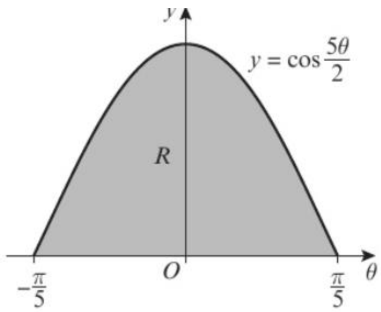
2 The diagram shows the region R bounded by the x-axis and the curve with equation $y = \cos\frac{5\theta}{2}$, $-\frac{\pi}{5} \leqslant \theta \leqslant \frac{\pi}{5}$

The table shows corresponding values of θ and y for $y = \cos\frac{5\theta}{2}$

θ	$-\frac{\pi}{5}$	$-\frac{\pi}{10}$	0	$\frac{\pi}{10}$	$\frac{\pi}{5}$
y	0		1		0



a Complete the table giving the missing values for y to 4 decimal places. **(1 mark)**

b Using the trapezium rule, with all the values for y in the completed table, find an approximation for the area of R, giving your answer to 3 decimal places. **(4 marks)**

c State, with a reason, whether your approximation in part **b** is an underestimate or an overestimate. **(1 mark)**

d Use integration to find the exact area of R. **(3 marks)**

e Calculate the percentage error in your answer in part **b**. **(2 marks)**

5 $f(x) = x^2 - \frac{3}{x^2}$, $x \geqslant 0$

a Show that a root α of the equation $f(x) = 0$ lies in the interval $[1.3, 1.4]$. **(1 mark)**

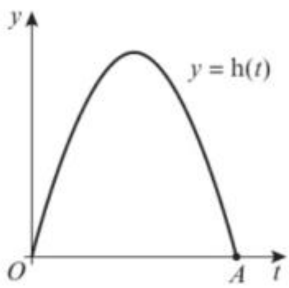
b Differentiate $f(x)$ to find $f'(x)$. **(2 marks)**

c By taking 1.3 as a first approximation to α, apply the Newton–Raphson process once to $f(x)$ to obtain a second approximation to α. Give your answer to 3 decimal places. **(3 marks)**

4 Ed throws a ball for his dog. The vertical height of the ball is modelled by the function

$$h(t) = 40\sin\left(\frac{t}{10}\right) - 9\cos\left(\frac{t}{10}\right) - 0.5t^2 + 9,\ t \geqslant 0$$

$y = h(t)$ is shown in the diagram.



a Show that the t-coordinate of A is the solution to

$$t = \sqrt{18 + 80\sin\left(\frac{t}{10}\right) - 18\cos\left(\frac{t}{10}\right)}$$

(3 marks)

To find an approximation for the t-coordinate of A, the iterative formula

$t_{n+1} = \sqrt{18 + 80\sin\left(\frac{t_n}{10}\right) - 18\cos\left(\frac{t_n}{10}\right)}$ is used.

b Let $t_0 = 8$. Find the values of t_1, t_2, t_3 and t_4. Give your answers to 3 decimal places. **(3 marks)**

c Find $h'(t)$. **(2 marks)**

d Taking 8 as a first approximation, apply the Newton–Raphson method once to $h(t)$ to obtain a second approximation for the time when the height of the ball is zero. Give your answer to 3 decimal places. **(3 marks)**

e Hence suggest an improvement to the range of validity of the model. **(2 marks)**

2 **a** 0.7071, 0.7071 **b** 0.758

c The shape of the graph is concave, so the trapezium lines will underestimate the area.

d 0.8 **e** 5.25%

5 **a** $f(1.3) = -0.085\ldots$, $f(1.4) = 0.429\ldots$ As there is a change of sign in the interval, there must be a root α in this interval.

b $f'(x) = 2x + \frac{6}{x^3}$ **c** 1.316

4 **a** $h(t) = 0$

$$40\sin\left(\frac{t}{10}\right) - 9\cos\left(\frac{t}{10}\right) - 0.5t^2 + 9 = 0$$

$$40\sin\left(\frac{t}{10}\right) - 9\cos\left(\frac{t}{10}\right) + 9 = 0.5t^2$$

$$80\sin\left(\frac{t}{10}\right) - 18\cos\left(\frac{t}{10}\right) + 18 = t^2$$

$$\Rightarrow t = \sqrt{18 + 80\sin\left(\frac{t}{10}\right) - 18\cos\left(\frac{t}{10}\right)}$$

b $t_1 = 7.928$, $t_2 = 7.896$, $t_3 = 7.882$, $t_4 = 7.876$

c $h'(t) = 4\cos\left(\frac{t}{10}\right) + 0.9\sin\left(\frac{t}{10}\right) - t$

d 7.874 (3 d.p.)

e Restrict the range of validity to $0 \leqslant t \leqslant A$