$$f(x) = \frac{1}{(ax+1)(1-bx)}$$

$$\frac{1}{(ax+1)(1-bx)} = \frac{A}{ax+1} + \frac{B}{1-bx}$$

$$\therefore\ 1 = A(1-bx) + B(ax+1)$$

$$x = \frac{1}{b} \Rightarrow 1 = B\left(\frac{a}{b}+1\right) \quad \therefore\ 1 = B\left(\frac{a+b}{b}\right) \quad \therefore\ B = \frac{b}{a+b}$$

$$x = -\frac{1}{a} \Rightarrow 1 = A\left(1+\frac{b}{a}\right) \quad \therefore\ 1 = A\left(\frac{a+b}{a}\right) \quad \therefore\ A = \frac{a}{a+b}$$

$$\therefore\ f(x) = \frac{a}{a+b}\left(\frac{1}{ax+1}\right) + \frac{b}{a+b}\left(\frac{1}{1-bx}\right)$$

$$\therefore \int f(x)\,dx = \int \frac{a}{a+b}\left(\frac{1}{ax+1}\right)dx + \int \frac{b}{a+b}\left(\frac{1}{1-bx}\right)dx$$

$$= \frac{1}{a+b}\int \frac{a}{ax+1}\,dx - \frac{1}{a+b}\int \frac{-b}{1-bx}\,dx$$

$$= \frac{1}{a+b}\ln|ax+1| - \frac{1}{a+b}\ln|1-bx| + c$$

$$= \frac{1}{a+b}\ln\left|\frac{ax+1}{1-bx}\right| + c$$