Cobweb and Staircase diagrams

1) a) i) Rearrange $2x^3 = 3x - 5$ in the form $x_{n+1} = \sqrt[3]{(Ax_n + B)}$
ii) Starting with $x_1 = 1$, find x_2, x_3, x_4, x_5, x_6 . Give each answer correct to 3 s.f.
iii) Draw a cobweb or staircase diagram to show whether this iteration converges to a root or diverges

1) b) i) Rearrange $2x^3 = 3x - 5$ in the form $x_{n+1} = Cx^3 + D$
ii) Starting with $x_1 = 1$, find x_2, x_3, x_4 Give each answer correct to 3 s.f.
iii) Draw a cobweb or staircase diagram to show whether this iteration converges to a root or diverges

1) c) i) Rearrange $2x^3 = 3x - 5$ in the form $x_{n+1} = \frac{E}{x^2} + \frac{F}{x^3}$
ii) Starting with $x_1 = 1$, find x_2, x_3, x_4 Give each answer correct to 3 s.f.
iii) Draw a cobweb or staircase diagram to show whether this iteration converges to a root or diverges

1d) Use your answers to 1a), 1b) and 1c) to solve the equation $2x^3 = 3x - 5$ correct to 3 s.f.

2 a) i) Rearrange $x^3 = 4x^2 - 1$ in the form $x_{n+1} = \sqrt[3]{(Ax_n^2 + B)}$
ii) Starting with $x_1 = 1$, find x_2, x_3, x_4. Give each answer correct to 3 s.f. Without writing down each intermediate term, write down x_{100} to 3 s.f.
iii) Draw a cobweb or staircase diagram to show whether this iteration converges to a root or diverges

2 b) i) Rearrange $x^3 = 4x^2 - 1$ in the form $x_{n+1} = \sqrt{(Cx_n^3 + D)}$
ii) Starting with $x_1 = 1$, find x_2, x_3, x_4. Give each answer correct to 3 s.f. Without writing down each intermediate term, write down x_{100} to 3 s.f.
iii) Draw a cobweb or staircase diagram to show whether this iteration converges to a root or diverges

2c) There is a third solution to $x^3 = 4x^2 - 1$ which is $x = -0.473$. Rearrange $x^3 = 4x^2 - 1$ in such a way as to find an iteration that converges to $x = -0.473$

2d) Use your answers to 2a), 2b) and 2c) to solve the equation $x^3 = 4x^2 - 1$ correct to 3 s.f.

1. a) Show that the equation $x^3 - 10x + 1 = 0$ can be arranged to the form $x = \frac{x^3+1}{10}$
b) Use this rearrangement to form an iterative formula and use it to find, correct to 4 s.f., the root that lies between 0 and 1.

2. Assuming the sequences defined by the following iterative formulae converge, find, in the form f(x) = 0 the equation each would solve.
$x_{n+1} = \frac{x_n^2+1}{4}$, $x_{n+1} = \frac{1}{5}(4x_n + \frac{50}{x_4^4})$, $x_{n+1} = \frac{\sin x_n - x_n + 2}{3 + x_n}$

3. a) The equation $x^3 - 5x - 2 = 0$ has a root between 2 and 3. Use the iterative formula
$x_{n+1} = \frac{2x_n^3+2}{3x_n2-5}$ starting with $x_1 = 2$, to find x_2, x_3, x_4 to 4 d.p.
b) Show that the root is 2.414 correct to 3 decimal places.
c) Hence find, correct to 2 d.p. a root of $2^{3x} - 5 \text{ x } 2^x - 2 = 0$