Question 1 (**)

$$x^3+10x-4=0.$$

a) Show that the above equation has a root α, which lies between 0 and 1.

The recurrence relation

$$x_{n+1}=\frac{4-x_n^3}{10}$$

starting with $x_0=0.3$ is to be used to find α.

b) Find, to 4 decimal places, the value of x_1, x_2, x_3 and x_4.

c) By considering the sign of an appropriate function $f(x)$ in a suitable interval, show clearly that $\alpha=0.39389$, correct to 5 decimal places.

Question 4 (**+)

$$f(x)=4x-3\sin x-1, \quad 0\le x\le 2\pi .$$

a) Show that the equation $f(x)=0$ has a solution α in the interval $(0.7,0.8)$.

An iterative formula, of the form given below, is used to find α.

$$x_{n+1}=A+B\sin x_n, \quad x_1=0.75,$$

where A and B are constants.

b) Find, to 5 decimal places, the value of x_2, x_3, x_4 and x_5.

c) By considering the sign of $f(x)$ in a suitable interval show clearly that $\alpha=0.775$, correct to 3 decimal places.

$\boxed{}$, $\boxed{x_2=0.76123,\ x_3=0.76736,\ x_4=0.77068,\ x_5=0.77247}$

Question 13 (***+)

The curve C has equation

$$y = x^3 - 3x^2 - 3,$$

and crosses the x axis at the point $A(\alpha, 0)$.

a) Show that α lies between 3 and 4.

b) Show further that the equation $x^3 - 3x^2 - 3 = 0$ can be rearranged to

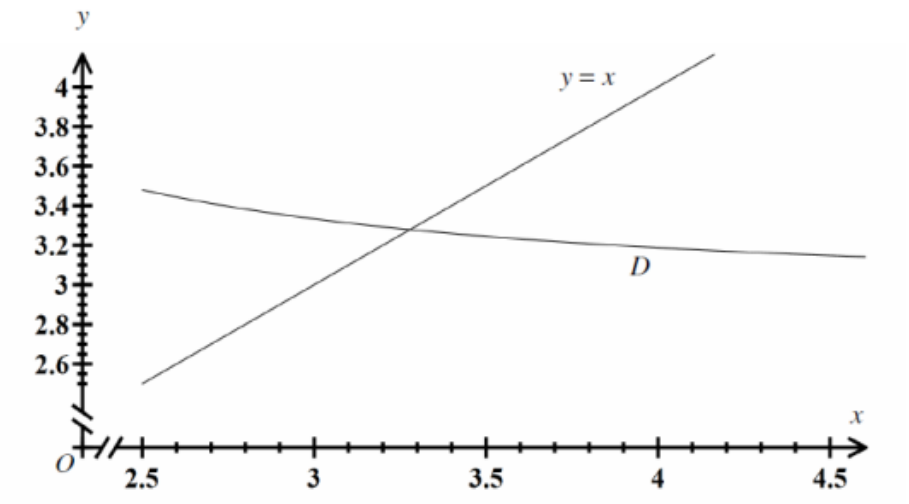
$$x = 3 + \frac{3}{x^2}, \quad x \neq 0.$$

The equation rearrangement of part **(b)** is written as the following recurrence relation

$$x_{n+1} = 3 + \frac{3}{x_n^2}, \quad x_1 = 4.$$

c) Use the above iterative formula to find, to 4 decimal places, the value of x_2, x_3, x_4 and x_5.

The diagram below is used to describe how the iteration formula converges to α, and shows the graph of $y = x$ and another curve D.



d) Write down the equation of D.

e) On a copy of the diagram draw a "staircase" or a "cob-web" pattern to show how the convergence to the root α is taking place, marking clearly the position of x_1, x_2 and x_3.

Question 9 (***)

A cubic equation has the following equation.

$$x^3+1=4x,\ x\in\mathbb{R}.$$

a) Show that the above equation has a root α, which lies between 0 and 1.

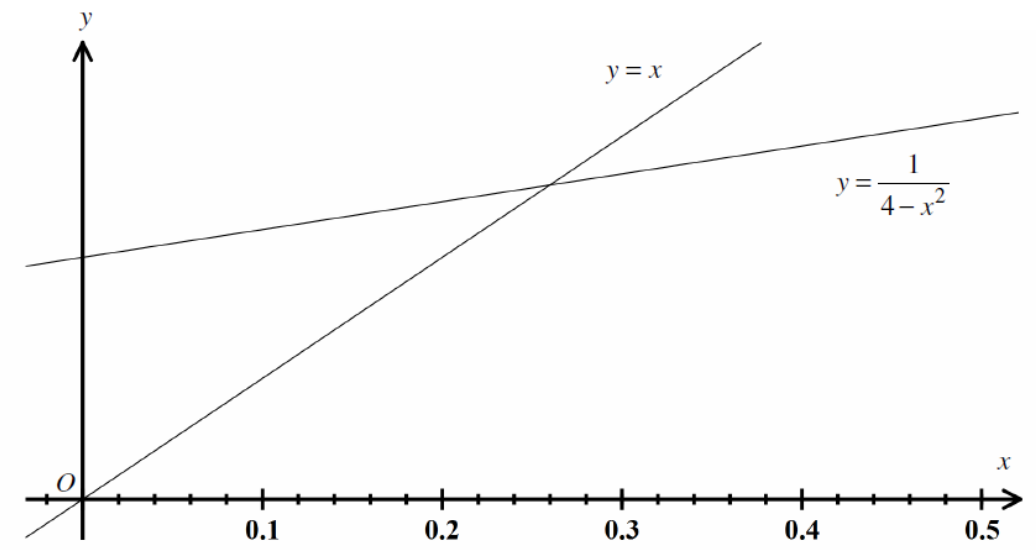
b) Show further that the above equation can be written as

$$x=\frac{1}{4-x^2}.$$

An iterative formula, based on the rearrangement of part (b), is to be used to find α.

c) Starting with $x_1=0.1$, find to 4 decimal places, the value of x_2, x_3 and x_4.

The diagram below is used to show the convergence of these iterations.



d) Draw on a copy of this diagram a "staircase" or "cobweb" pattern showing how these iterations converge to α, marking the position of x_1, x_2, x_3 and x_4.

Question 16 (****)

A non uniform plank AB has length 12 m and mass M kg.

A smooth support is placed under the plank at the point C, where $|AC| = 3$ m. When a child of mass 30 kg stands at A, the plank rest horizontally in equilibrium.

The smooth support is next placed under the plank at the point D, where $|BD| = 5$ m. When the same child stands at B, the plank again rest horizontally in equilibrium.

The plank is modelled as a non uniform rod whose centre of mass is at the point G, and the child is modelled as a particle.

a) Determine the value of M.

b) Calculate the distance AG.

Two smooth supports are next placed under the plank at the points C and D, and when the same child stands at E, the plank rest horizontally in equilibrium with the reactions at the two supports being equal.

c) Find the distance AE.

Question 6

The equation of a curve is given by

$$x^2 - 2y^2 - xy - x + 5y + 34 = 0.$$

a) Show clearly that

$$\frac{dy}{dx} = \frac{2x - y - 1}{x + 4y - 5}.$$

b) Find the exact value of gradient at the point on the curve with coordinates

$$\left(1 + 4\sqrt{2}, -5 - \sqrt{2}\right).$$

c) Determine the coordinates of the turning point of the curve.

1) $\boxed{x_1 = 0.3973,\quad x_2 = 0.3937,\quad x_3 = 0.3939,\quad x_4 = 0.3939}$

4) $\boxed{x_2 = 0.76123,\ x_3 = 0.76736,\ x_4 = 0.77068,\ x_5 = 0.77247}$

13) $\boxed{x_1 = 3.1875,\quad x_2 = 3.2953,\quad x_3 = 3.2763,\quad x_4 = 3.2794}$,

$\boxed{D: y = 3 + \frac{3}{x^2}}$

9) $\boxed{x_2 = 0.2506,\quad x_3 = 0.2540,\quad x_4 = 0.2541}$

16) , $\boxed{M = 60}$, $\boxed{|AG| = 4.5\ \text{m}}$, $\boxed{|AE| = 6\ \text{m}}$

6) $\boxed{-\frac{1}{8}\left(2 + 3\sqrt{2}\right)}$, $\boxed{(3,5), (-1,-3)}$