Questions from the June 2005 papers

1. The function f is defined by f: $x \mapsto \dfrac{5x+1}{x^2+x-2} - \dfrac{3}{x+2}$, $x > 1$.

(*a*) Find f(x) =, $x > 1$.

(*b*) Find $f^{-1}(x)$.

The function g is defined by g: $x \mapsto x^2 + 5$, $x \in \mathbb{R}$.

(*c*) Solve fg(x) = $\frac{1}{4}$.

2. A curve has equation $x^2 + 2xy - 3y^2 + 16 = 0$. Find the coordinates of the points on the curve where $\dfrac{dy}{dx} = 0$.

3. Use the binomial theorem to expand $\sqrt{(4 - 9x)}$, $|x| < \dfrac{4}{9}$, in ascending powers of x, up to and including the term in x^3, simplifying each term.

4. (*a*) Express $\dfrac{5x+3}{(2x-3)(x+2)}$ in partial fractions.

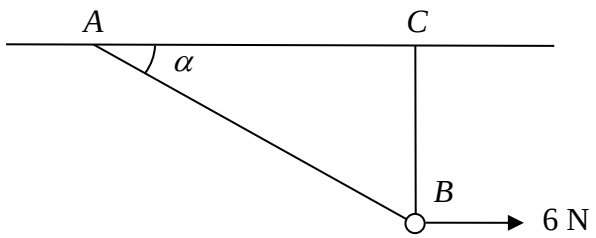
(*b*) Hence find the exact value of $\displaystyle\int_2^6 \frac{5x+3}{(2x-3)(x+2)}\,dx$, giving your answer as a single logarithm.

5. In taking off, an aircraft moves on a straight runway *AB* of length 1.2 km. The aircraft moves from *A* with initial speed 2 m s^{-1}. It moves with constant acceleration and 20 s later it leaves the runway at *C* with speed 74 m s^{-1}. Find

(*a*) the acceleration of the aircraft,

(*b*) the distance *BC*.

6. Figure 1

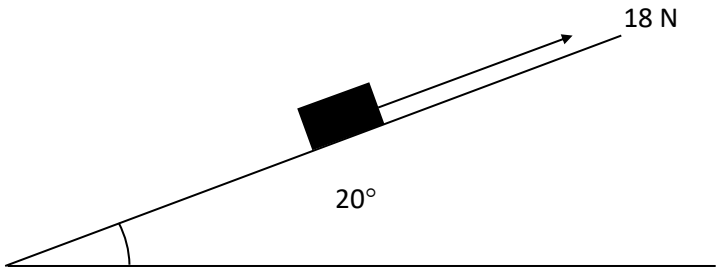


A smooth bead *B* is threaded on a light inextensible string. The ends of the string are attached to two fixed points *A* and *C* on the same horizontal level. The bead is held in equilibrium by a horizontal force of magnitude 6 N acting parallel to *AC*. The bead *B* is vertically below *C* and $\angle BAC = \alpha$, as shown in Figure 1. Given that $\tan \alpha = \frac{3}{4}$, find

(*a*) the tension in the string,

(*b*) the weight of the bead.

7. Figure 2



A box of mass 2 kg is pulled up a rough plane face by means of a light rope. The plane is inclined at an angle of 20° to the horizontal, as shown in Figure 2. The rope is parallel to a line of greatest slope of the plane. The tension in the rope is 18 N. The coefficient of friction between the box and the plane is 0.6. By modelling the box as a particle, find

(*a*) the normal reaction of the plane on the box,

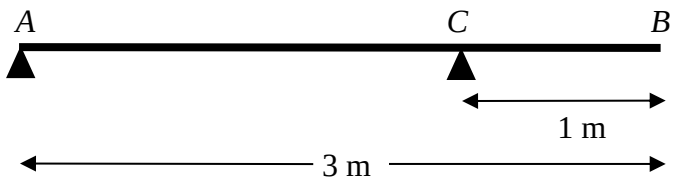
(*b*) the acceleration of the box.

8. A train is travelling at 10 m s^{-1} on a straight horizontal track. The driver sees a red signal 135 m ahead and immediately applies the brakes. The train immediately decelerates with constant deceleration for 12 s, reducing its speed to 3 m s^{-1}. The driver then releases the brakes and allows the train to travel at a constant speed of 3 m s^{-1} for a further 15 s. He then applies the brakes again and the train slows down with constant deceleration, coming to rest as it reaches the signal.

(a) Find the distance travelled by the train from the moment when the brakes are first applied to the moment when its speed first reaches 3 m s^{-1}.

(b) Find the total time from the moment when the brakes are first applied to the moment when the train comes to rest.

9. Figure 3



A uniform beam *AB* has mass 12 kg and length 3 m. The beam rests in equilibrium in a horizontal position, resting on two smooth supports. One support is at end *A*, the other at a point *C* on the beam, where *BC* = 1 m, as shown in Figure 3. The beam is modelled as a uniform rod.

(*a*) Find the reaction on the beam at *C*.

A woman of mass 48 kg stands on the beam at the point *D*. The beam remains in equilibrium. The reactions on the beam at *A* and *C* are now equal.

(*b*) Find the distance *AD*.

10. A Maths professor of my acquaintance has noticed that the long number N on her credit card has 10 digits. The first digit of N is the number of zeros in N, the second digit is the number of ones, the third digit is the number of twos, and so on until the last digit is the number of 9s in N. What is her credit card number?