

Sigma Notation

For each series:

write out every term in the series
hence find the value of the sum.

(1, 2) $\sum_{r=1}^5 (3r+1)$

(3, 4) $\sum_{r=1}^6 3r^2$

(5, 6) $\sum_{r=1}^5 \sin(90r^\circ)$

Recurrence Relations

Find the first four terms of the following recurrence relationships.

(7) $u_{n+1} = u_n + 3, u_1 = 1$

(8) $u_{n+1} = u_n - 5, u_1 = 9$

(9) $u_{n+1} = 2u_n + 1, u_1 = 2$

(10) $u_{n+1} = (u_n)^2 - 1, u_1 = 2$

Suggest possible recurrence relationships for the following sequences. (Remember to state the first term.)

(11) 20, 17, 14, 11, ...

(12) 1, 2, 4, 8, ...

(13) 1, -1, 1, -1, 1, ...

(14) 3, 7, 15, 31, ...

(15) 0, 1, 2, 5, 26, ...

(16) 26, 14, 8, 5, 3.5, ...

By writing down the first four terms or otherwise, find the recurrence formula that defines the following sequences:

(17) $u_n = 2n - 1$

(18) $u_n = 3n + 2$

(19) $u_n = n + 2$

(20) $u_n = n^2$

(21) $u_n = 3^n - 1$

(22, 23) A sequence is defined for $n \geq 1$ by the recurrence relation

$$u_{n+1} = pu_n + q, u_1 = 2$$

Given that $u_2 = -1$ and $u_3 = 11$, find the values of p and q .

A sequence is given by

$$x_1 = 2$$

$$x_{n+1} = x_n(p - 3x_n)$$

where p is an integer.

(24) Show that $x_3 = -10p^2 + 132p - k$ state k

(25) Given that $x_3 = -288$ find the value of p .

(26) Hence find the value of x_4 .

(29) $\sum_{r=2}^5 (r^2 + k) = 65$
state k .

(30) $\sum_{r=1}^3 pr^3 = 108$
state p

(31) $\sum_{r=2}^6 ar^2 + br^3 = 1145$
state $\frac{b}{a}$

A sequence a_1, a_2, a_3, \dots is defined by

$$a_1 = k$$

$$a_{n+1} = 4a_n + 5$$

(27) Find a_3 in terms of k .

(28) Show that $\sum_{r=1}^4 a_r$ is a multiple of p where p is a prime number

- A $u_{n+1} = 2u_n, u_1 = 1$
- B $u_{n+1} = \frac{1}{2}(u_n + 2), u_1 = 26$
- C 1
- D $4 + 7 + 10 + 13 + 16$
- E $16k + 25$
- F $u_{n+1} = u_n + 2n + 1, u_1 = 1$
- G $u_{n+1} = -u_n, u_1 = 1$
- H 2, 3, 8, 63
- I 1, 4, 7, 10
- J -4
- K 432
- L 2, 5, 11, 23
- M 50
- N 9, 4, -1, -6
- O $u_{n+1} = u_n - 3, u_1 = 20$
- P $u_{n+1} = 3u_n + 2, u_1 = 2$
- Q $3 + 12 + 27 + 48 + 75 + 108$
- R $u_{n+1} = (u_n)^2 + 1, u_1 = 0$
- S 1.5
- T $u_{n+1} = u_n + 1, u_1 = 3$
- U $u_{n+1} = u_n + 2, u_1 = 1$
- V 273
- W $u_{n+1} = 2u_n + 1$
- X 12
- Y 2
- Z $u_{n+1} = u_n + 3, u_1 = 5$
- α $1 + 0 + (-1) + 0 + 1$
- β 7
- δ -252288
- ς 5
- ε 3

