Exercise 6E

In this exercise, all angles are given in radians.

1 Without using a calculator, work out, giving your answer in terms of π:

a $\arccos(0)$ b $\arcsin(1)$ c $\arctan(-1)$ d $\arcsin(-\frac{1}{2})$

e $\arccos\left(-\frac{1}{\sqrt{2}}\right)$ f $\arctan\left(-\frac{1}{\sqrt{3}}\right)$ g $\arcsin\left(\sin\frac{\pi}{3}\right)$ h $\arcsin\left(\sin\frac{2\pi}{3}\right)$

2 Find:

a $\arcsin(\frac{1}{2}) + \arcsin(-\frac{1}{2})$ b $\arccos(\frac{1}{2}) - \arccos(-\frac{1}{2})$ c $\arctan(1) - \arctan(-1)$

(P) 3 Without using a calculator, work out the values of:

a $\sin(\arcsin\frac{1}{2})$ b $\sin(\arcsin(-\frac{1}{2}))$

c $\tan(\arctan(-1))$ d $\cos(\arccos 0)$

(P) 4 Without using a calculator, work out the exact values of:

a $\sin(\arccos(\frac{1}{2}))$ b $\cos(\arcsin(-\frac{1}{2}))$ c $\tan\left(\arccos\left(-\frac{\sqrt{2}}{2}\right)\right)$

d $\sec(\arctan(\sqrt{3}))$ e $\operatorname{cosec}(\arcsin(-1))$ f $\sin\left(2\arcsin\left(\frac{\sqrt{2}}{2}\right)\right)$

(P) 5 Given that $\arcsin k = \alpha$, where $0 < k < 1$ and α is in radians, write down, in terms of α, the first two positive values of x satisfying the equation $\sin x = k$.

(P) 6 Given that x satisfies $\arcsin x = k$, where $0 < k < \frac{\pi}{2}$,

a state the range of possible values of x **(1 mark)**

b express, in terms of x,

i $\cos k$ ii $\tan k$ **(4 marks)**

Given, instead, that $-\frac{\pi}{2} < k < 0$,

c how, if at all, are your answers to part **b** affected? **(2 marks)**

(P) 7 Sketch the graphs of:

a $y = \frac{\pi}{2} + 2\arcsin x$ b $y = \pi - \arctan x$

c $y = \arccos(2x + 1)$ d $y = -2\arcsin(-x)$

(P) 8 The function f is defined as $f: x \to \arcsin x$, $-1 \leqslant x \leqslant 1$, and the function g is such that $g(x) = f(2x)$.

a Sketch the graph of $y = f(x)$ and state the range of f. **(3 marks)**

b Sketch the graph of $y = g(x)$. **(2 marks)**

c Define g in the form $g: x \mapsto \ldots$ and give the domain of g. **(3 marks)**

d Define g^{-1} in the form $g^{-1}: x \mapsto \ldots$ **(2 marks)**

(P) 9 a Prove that for $0 \leqslant x \leqslant 1$, $\arccos x = \arcsin\sqrt{1 - x^2}$ **(4 marks)**

b Give a reason why this result is not true for $-1 \leqslant x \leqslant 0$. **(2 marks)**

Exercise 6E

1 a $\frac{\pi}{2}$ b $\frac{\pi}{2}$ c $-\frac{\pi}{4}$ d $-\frac{\pi}{6}$

e $\frac{3\pi}{4}$ f $-\frac{\pi}{6}$ g $\frac{\pi}{3}$ h $\frac{\pi}{3}$

2 a 0 b $-\frac{\pi}{3}$ c $\frac{\pi}{2}$

3 a $\frac{1}{2}$ b $-\frac{1}{2}$ c -1 d 0

4 a $\frac{\sqrt{3}}{2}$ b $\frac{\sqrt{3}}{2}$ c -1 d 2

e -1 f 1

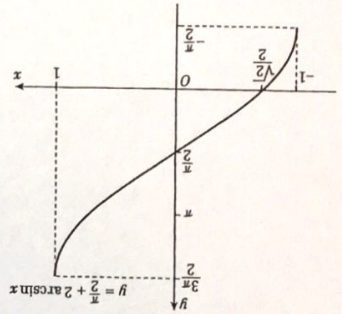
5 $\alpha, \pi - \alpha$

6 a $0 < x < 1$

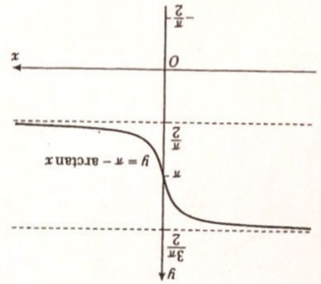
b i $\sqrt{1-x^2}$ ii $\frac{x}{\sqrt{1-x^2}}$

c i no change ii no change

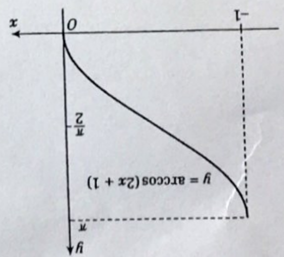
7 a



b



c



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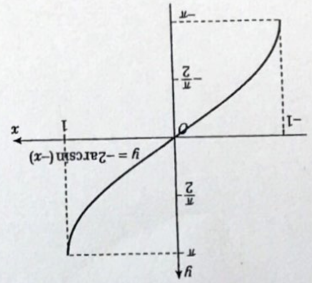
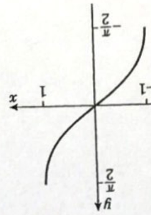
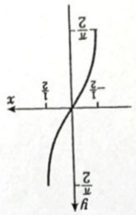


8 a



Range: $-\frac{\pi}{2} \leqslant f(x) \leqslant \frac{\pi}{2}$

b



c $g: x \to \arcsin 2x$, $-\frac{1}{2} \leqslant x \leqslant \frac{1}{2}$

d $g^{-1}: x \to \frac{1}{2}\sin x$, $-\frac{\pi}{2} \leqslant x \leqslant \frac{\pi}{2}$

9 a Let $y = \arccos x$, $x \in [0,1] \Rightarrow y \in [0, \frac{\pi}{2}]$

$\cos y = x$, so $\sin y = \sqrt{1 - \cos^2 y} = \sqrt{1 - x^2}$

(Note, $\sin y \neq -\sqrt{1 - x^2}$ since $y \in [0, \frac{\pi}{2}]$, so $\sin y \geqslant 0$)

$y = \arcsin\sqrt{1 - x^2}$

Therefore, $\arccos x = \arcsin\sqrt{1 - x^2}$ for $x \in [0,1]$.

b For $x \in (-1, 0)$, $\arccos x \in (\frac{\pi}{2}, \pi)$, but arcsin is only has range $[-\frac{\pi}{2}, \frac{\pi}{2}]$.