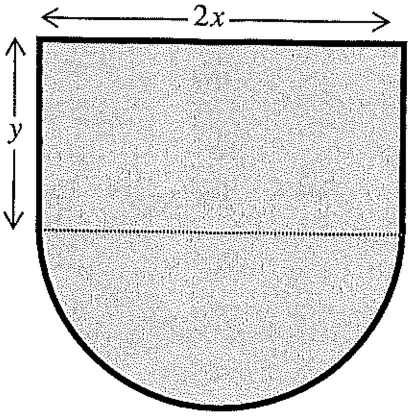
①



The figure above shows the design of a theatre stage which is the shape of a semicircle attached to rectangle. The diameter of the semicircle is $2x$ m and is attached to one side of the rectangle also measuring $2x$ m. The other side of the rectangle is y m.

The perimeter of the stage is 60 m.

a) Show that the total area of the stage, A m^2, is given by

$$A = ax - bx^2 - \frac{1}{c}\pi x^2.$$ State a, b, c

b) Determine by differentiation an **exact** value of x for which A is stationary.

c) Show that the value of x found in part **(b)** gives the maximum or minimum? value for A.

d) Show **clearly** that the maximum area of the stage is $\frac{d}{\pi + 4}$ m^2.

②

The curve C has equation

$$y = 2^{x-3}.$$

a) Describe geometrically a single transformation that maps the graph of $y = 2^x$ onto the graph of C.

b) Describe geometrically a different transformation that can also map the graph of $y = 2^x$ onto the graph of C.

③

$$\int \frac{6}{(3x+1)^3}\,dx$$

④

$$\int \cos\tfrac{1}{4}x + \sin\tfrac{1}{2}x\,dx$$

5

$$f(x) = \sqrt{x}, \ x \in \mathbb{R}, \ x \geq 0.$$

The graph of $f(x)$ is translated by 3 units in the negative x direction, followed by a reflection in the y axis, forming the graph of $g(x)$.

a) Find the equation of $g(x)$.

b) Sketch the graph of $g(x)$.

The sketch must include the coordinates of all the points where the curve meets the coordinate axes.

6 $\displaystyle\int \frac{\cos(\ln x)}{x} \, dx$

7 In the convergent expansion of

$$\left(1 + \tfrac{4}{7}nx\right)^n, \ n \in \mathbb{R}, \ n \notin \mathbb{N}, \ n \neq 0,$$

the coefficients of x^2 and x^3 are non zero and equal.

a) Determine the possible values of n.

b) State with justification which value, values or indeed if any of the values of n produces a valid expansion for $x = 1$.

8 $\displaystyle\int 2\cos 2x - \sin\tfrac{x}{2} + 6\sin\tfrac{2x}{3} \, dx$

9

$$f(x) = \sqrt{\frac{1+ax}{4-x}}, \ -1 < x < 1.$$

The value of the constant a is such so that the coefficient of x^2 in the convergent binomial expansion of $f(x)$ is $\dfrac{1}{64}$.

Find the value of a.

10

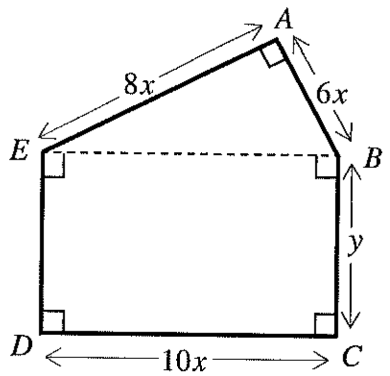
$$f(x) = x^2 + 6x + 10, \; x \in \mathbb{R}.$$

a) Express $f(x)$ in the form $f(x) = (x+a)^2 + b$, where a and b are integers.

b) Describe geometrically the transformation which map the graph of x^2 onto the graph of $f(x)$.

11

The figure below shows a pentagon $ABCDE$ whose measurements, in cm, are given in terms of x and y.



a) If the perimeter of the pentagon is 120 cm, show clearly that its area, A cm^2, is given by

$$A = a\,x - b\,x^2.$$ State a, b

b) Find, by differentiation, the value of x for which A is stationary.

c) Calculate the maximum value for A, fully justifying the fact that it is indeed the maximum value of A.

12

$$\int \sin 2x \cos 2x \, dx$$

① a) $A = 30$ $B = 60$ $C = 120$ $D = 15$

$A = 0$ $B = 1$ $C = 2$ $D = 4$

$A = 1$ $B = 2$ $C = 2$ $D = 4$

b) $A = \frac{60}{\pi + 4}$ $B = \frac{\pi + 4}{60}$ $C = \frac{60}{\pi} + 4$ $D = \frac{\pi}{60} + 4$

c) A = Maximum B = Minimum

d) $A = 1$ $B = 2$ $C = 2$ $D = 4$

② a) A = Translation right 3 units B = Translation left 3 units

C = Translation up 3 units D = Translation down 3 units

b) A = one way stretch vertically, scale factor $1/3$

B = one way stretch vertically, scale factor 3

C = one way stretch vertically, scale factor $1/18$

D = one way stretch vertically, scale factor 8

③ $A = 6\ln(3x+1)^3 + c$ $B = \frac{1}{(3x+1)^2} + c$ $C = \frac{-3}{(3x+1)^2} + c$

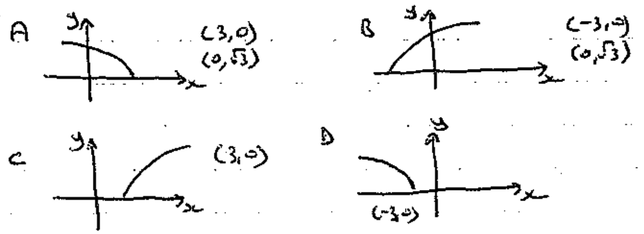
$D = \frac{2}{3}\ln(3x+1)^3 + c$

④ $A = \frac{1}{4}\sin\frac{1}{4}x + 2\cos\frac{1}{2}x + c$ $B = 4\sin\frac{1}{4}x - 2\cos\frac{1}{2}x + c$

$C = 4\sin\frac{1}{4}x + 2\cos\frac{1}{2}x + c$ $D = \frac{1}{4}\sin\frac{1}{4}x - 2\cos\frac{1}{2}x + c$

⑤ a) $A = 3 - x$ $B = \sqrt{x} - 3$ $C = \sqrt{3 + x}$ $D = \sqrt{3} - x$

b)



⑥ A $\frac{\sin(\ln x)}{x} + c$ B $\cos(\ln x) + c$ C $\sin(\ln x) + c$

D $-\frac{\cos(\ln x)}{x^2} + c$

⑦ a) $A = -3/2, -7/2$ $B = \frac{3}{2}, \frac{7}{2}$ $C = -3/2, 7/2$ $D = 3/2, -7/2$

b) $A = -3/2$ $B = 7/2$ $C = 3/2$ $D = -7/2$

⑧ $A = \sin 2x - 2\cos\frac{x}{2} + 9\cos\frac{2x}{3} + c$

$B = 2\sin 2x - \cos\frac{x}{2} - 9\cos\frac{2x}{3} + c$

$C = \sin 2x + \cos\frac{x}{2} + 3\cos\frac{2x}{3} + c$

$D = 2\sin 2x + 2\cos\frac{x}{2} - 3\cos\frac{2x}{3} + c$

⑨ $A = 1$ $B = 2$ $C = 4$ $D = 8$

⑩ a) $A = 1$ $B = 2$ $C = 4$ $D = 8$

$A = 1$ $B = 2$ $C = 4$ $D = 8$

b) A = translation $\binom{1}{2}$ B = translation $\binom{1}{3}$

C = translation $\binom{1}{0}$ D = translation $\binom{-1}{2}$

⑪ a) $A = 120$ $B = 240$ $C = 360$ $D = 600$

$A = 16$ $B = 96$ $C = 192$ $D = 384$

b) $A = \frac{25}{4}$ $B = \frac{25}{2}$ $C = \frac{25}{8}$ $D = 25$

c) $A = 937.5$ $B = 126.5$ $C = 1024.5$ $D = 714.5$

⑫ $A = \sin^2 2x + c$ $B = \frac{1}{2}\sin^2 2x + c$ $C = \frac{1}{4}\sin^2 2x + c$

$D = -\frac{2}{3}\sin^2 2x + c$ $E = -\frac{1}{2}\sin^2 2x + c$