Newton Raphson. method

1

$f(x) = \ln(3x - 4) - x^2 + 10,\ x > \frac{4}{3}$

a Show that $f(x) = 0$ has a root α in the interval [3.4, 3.5]. **(2 marks)**

b Find $f'(x)$. **(2 marks)**

c Taking 3.4 as a first approximation to α, apply the Newton–Raphson procedure once to $f(x)$ to obtain a second approximation for α, giving your answer to 3 decimal places. **(3 marks)**

2

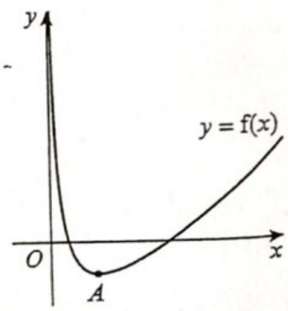
The diagram shows part of the curve with equation

$y = f(x)$, where $f(x) = x^{\frac{3}{2}} - e^{-x} + \dfrac{1}{\sqrt{x}} - 2,\ x > 0$.

The point A, with x-coordinate q, is a stationary point on the curve.
The equation $f(x) = 0$ has a root α in the interval [1.2, 1.3].

a Explain why $x_0 = q$ is not suitable to use as a first approximation when applying the Newton–Raphson method. **(1 mark)**

b Taking $x_0 = 1.2$ as a first approximation to α, apply the Newton–Raphson process once to $f(x)$ to obtain a second approximation to α. Give your answer to 3 decimal places. **(4 marks)**



3

$f(x) = 1 - x - \cos(x^2)$

a Show that the equation $f(x) = 0$ has a root α in the interval $1.4 < \alpha < 1.5$. **(1 mark)**

b Using $x_0 = 1.4$ as a first approximation to α, apply the Newton–Raphson procedure once to $f(x)$ to find a second approximation to α, giving your answer to 3 decimal places. **(4 marks)**

c By considering the change of sign of $f(x)$ over an appropriate interval, show that your answer to part **b** is correct to 3 decimal places. **(2 marks)**

4

$y = f(x)$, where $f(x) = x^2 \sin x - 2x + 1$. The points P, Q, and R are roots of the equation. The points A and B are stationary points, with x-coordinates a and b respectively.

a Show that the curve has a root in each of the following intervals:

~~**i** [0.6, 0.7]~~ **(1 mark)**

~~**ii** [1.2, 1.3]~~ Just do iii) **(1 mark)**

iii [2.4, 2.5] **(1 mark)**

b Explain why $x_0 = a$ is not suitable to use as a first approximation to α when applying the Newton–Raphson method to $f(x)$. **(1 mark)**

c Using $x_0 = 2.4$ as a first approximation, apply the Newton–Raphson method to $f(x)$ to obtain a second approximation. Give your answer to 3 decimal places. **(4 marks)**

Words !!! Yikes !!!

5

The price of a car in £s, x years after purchase, is modelled by the function

$$f(x) = 15\,000\,(0.85)^x - 1000 \sin x,\ x > 0$$

a Use the model to find the value, to the nearest hundred £s, of the car 10 years after purchase.

b Show that f(x) has a root between 19 and 20.

c Find f′(x).

d Taking 19.5 as a first approximation, apply the Newton–Raphson method once to f(x) to obtain a second approximation for the time when the value of the car is zero. Give your answer to 3 decimal places.

e Criticise this model with respect to the value of the car as it gets older.

6

An astronomer is studying the motion of a planet moving along an elliptical orbit. She formulates the following model relating the angle moved at a given time, E radians, to the angle the planet would have moved if it had been travelling on a circular path, M radians:

$$M = E - 0.1 \sin E,\ E \geqslant 0$$

In order to predict the position of the planet at a particular time, the astronomer needs to find the value of E when $M = \frac{\pi}{6}$

a Show that this value of E is a root of the function $f(x) = x - 0.1 \sin x - k$ where k is a constant to be determined.

b Taking 0.6 as a first approximation, apply the Newton–Raphson procedure once to f(x) to obtain a second approximation for the value of E when $M = \frac{\pi}{6}$

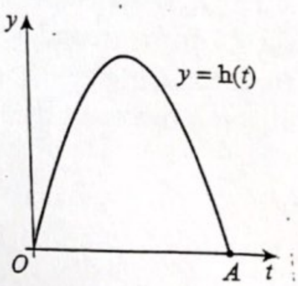
c By considering a change of sign on a suitable interval of f(x), show that your answer to part **b** is correct to 3 decimal places.

7

Ed throws a ball for his dog. The vertical height of the ball is modelled by the function

$$h(t) = 40 \sin\left(\frac{t}{10}\right) - 9 \cos\left(\frac{t}{10}\right) - 0.5t^2 + 9,\ t \geqslant 0$$

$y = h(t)$ is shown in the diagram.



a Show that the t-coordinate of A is the solution to

$$t = \sqrt{18 + 80 \sin\left(\frac{t}{10}\right) - 18 \cos\left(\frac{t}{10}\right)}$$

(3 marks)

Do parts c) and d) BEFORE parts a) and b) !!!! then e)

To find an approximation for the t-coordinate of A, the iterative formula

$$t_{n+1} = \sqrt{18 + 80 \sin\left(\frac{t_n}{10}\right) - 18 \cos\left(\frac{t_n}{10}\right)} \text{ is used.}$$

b Let $t_0 = 8$. Find the values of t_1, t_2, t_3 and t_4. Give your answers to 3 decimal places. **(3 marks)**

c Find h′(t). **(2 marks)**

d Taking 8 as a first approximation, apply the Newton–Raphson method once to h(t) to obtain a second approximation for the time when the height of the ball is zero. Give your answer to 3 decimal places. **(3 marks)**

e Hence suggest an improvement to the range of validity of the model. **(2 marks)**