1 For each of the following,

i find the binomial expansion up to and including the x^3 term

ii state the range of values of x for which the expansion is valid.

a $\sqrt{4 + 2x}$ **b** $\dfrac{1}{2 + x}$ **c** $\dfrac{1}{(4 - x)^2}$

2 $f(x) = (5 + 4x)^{-2}$, $|x| < \dfrac{5}{4}$

Find the binomial expansion of $f(x)$ in ascending powers of x, up to and including the term in x^3. Give each coefficient as a simplified fraction. **(5 marks)**

3 $m(x) = \sqrt{4 - x}$, $|x| < 4$

a Find the series expansion of $m(x)$, in ascending powers of x, up to and including the x^2 term. Simplify each term. **(4 marks)**

b Show that, when $x = \dfrac{1}{9}$, the exact value of $m(x)$ is $\dfrac{\sqrt{35}}{3}$ **(2 marks)**

c Use your answer to part **a** to find an approximate value for $\sqrt{35}$, and calculate the percentage error in your approximation. **(4 marks)**

4 The first three terms in the binomial expansion of $\dfrac{1}{\sqrt{a + bx}}$ are $3 + \dfrac{1}{3}x + \dfrac{1}{18}x^2 + \ldots$

a Find the values of the constants a and b.

b Find the coefficient of the x^3 term in the expansion.

5 $f(x) = \dfrac{3 + 2x - x^2}{4 - x}$

Prove that if x is sufficiently small, $f(x)$ may be approximated by $\dfrac{3}{4} + \dfrac{11}{16}x - \dfrac{5}{64}x^2$.

1 **a** **i** $2+\frac{x}{2}-\frac{x^2}{16}+\frac{x^3}{64}$ **ii** $|x|<2$

b **i** $\frac{1}{2}-\frac{x}{4}+\frac{x^2}{8}-\frac{x^3}{16}$ **ii** $|x|<2$

c **i** $\frac{1}{16}+\frac{x}{32}-\frac{3x^2}{256}+\frac{x^3}{256}$ **ii** $|x|<4$

2 $\frac{1}{25}-\frac{8}{125}x+\frac{48}{625}x^2-\frac{256}{3125}x^3$

3 **a** $2-\frac{x}{4}-\frac{x^2}{64}$

b $\mathrm{m}(x)=\sqrt{\frac{35}{9}}=\frac{\sqrt{35}}{\sqrt{9}}=\frac{\sqrt{35}}{3}$

c 5.91609 (correct to 5 decimal places),
% error $= 1.38\times 10^{-4}\%$

4 **a** $a=\frac{1}{9}, b=-\frac{2}{81}$ **b** $\frac{5}{486}$

5 For small values of x ignore powers of x^3 and higher.

$(4-x)^{-1}=\frac{1}{4}+\frac{x}{16}+\frac{x^2}{64}+\ldots$

Multiply by $(3+2x-x^2)=\frac{3}{4}+\frac{x}{2}-\frac{x^2}{4}+\frac{3x}{16}+\frac{x^2}{8}+\frac{3x^2}{64}$

$=\frac{3}{4}+\frac{11}{16}x-\frac{5}{64}x^2$