

① (i) Expand  $(1 + ax)^{-4}$  in ascending powers of  $x$ , up to and including the term in  $x^2$ . [3]

(ii) The coefficients of  $x$  and  $x^2$  in the expansion of  $(1 + bx)(1 + ax)^{-4}$  are 1 and  $-2$  respectively. Given that  $a > 0$ , find the values of  $a$  and  $b$ . [5]

② Find the first three terms in the expansion of  $(9 - 16x)^{\frac{3}{2}}$  in ascending powers of  $x$ , and state the set of values for which this expansion is valid. [5]

③ (i) Expand  $(1 + 2x)^{\frac{1}{2}}$  as a series in ascending powers of  $x$ , up to and including the term in  $x^3$ . [3]

(ii) Hence find the expansion of  $\frac{(1 + 2x)^{\frac{1}{2}}}{(1 + x)^3}$  as a series in ascending powers of  $x$ , up to and including the term in  $x^3$ . [5]

(iii) State the set of values of  $x$  for which the expansion in part (ii) is valid. [1]

④ (i) Expand  $(1 - x)^{\frac{1}{2}}$  in ascending powers of  $x$  as far as the term in  $x^2$ . [3]

(ii) Hence expand  $(1 - 2y + 4y^2)^{\frac{1}{2}}$  in ascending powers of  $y$  as far as the term in  $y^2$ . [3]

⑤ (i) Expand  $(1 + x)^{\frac{1}{3}}$  in ascending powers of  $x$ , up to and including the term in  $x^2$ . [2]

(ii) (a) Hence, or otherwise, expand  $(8 + 16x)^{\frac{1}{3}}$  in ascending powers of  $x$ , up to and including the term in  $x^2$ . [4]

(b) State the set of values of  $x$  for which the expansion in part (ii) (a) is valid. [1]

Handwritten solutions for the problems above:

① (i)  $(1 + ax)^{-4} = 1 - 4ax + \frac{(-4)(-5)}{2} a^2 x^2 + \dots = 1 - 4ax + 10a^2 x^2 + \dots$

(ii)  $(1 + bx)(1 + ax)^{-4} = (1 + bx)(1 - 4ax + 10a^2 x^2 + \dots)$   
 $= 1 + bx - 4ax - 4abx^2 + 10a^2 x^2 + \dots$   
 $= 1 + (b - 4a)x + (10a^2 - 4ab)x^2 + \dots$   
 Coefficient of  $x$  is 1:  $b - 4a = 1$  (1)  
 Coefficient of  $x^2$  is  $-2$ :  $10a^2 - 4ab = -2$  (2)

②  $(9 - 16x)^{\frac{3}{2}} = 9^{\frac{3}{2}} (1 - \frac{16}{9}x)^{\frac{3}{2}} = 27 (1 - \frac{16}{9}x)^{\frac{3}{2}}$   
 $= 27 [1 - \frac{3}{2}(\frac{16}{9}x) + \frac{(-3/2)(-5/2)}{2} (\frac{16}{9}x)^2 + \dots]$   
 $= 27 [1 - 2x + \frac{10}{9}(\frac{256}{81})x^2 + \dots]$   
 $= 27 [1 - 2x + \frac{256}{81}x^2 + \dots]$   
 $= 27 - 54x + \frac{256}{3}x^2 + \dots$

③ (i)  $(1 + 2x)^{\frac{1}{2}} = 1 + \frac{1}{2}(2x) - \frac{1}{8}(2x)^2 + \frac{1}{16}(2x)^3 + \dots$   
 $= 1 + x - \frac{1}{2}x^2 + \frac{1}{2}x^3 + \dots$

(ii)  $\frac{(1 + 2x)^{\frac{1}{2}}}{(1 + x)^3} = (1 + x - \frac{1}{2}x^2 + \frac{1}{2}x^3 + \dots)(1 + x)^{-3}$   
 $= (1 + x - \frac{1}{2}x^2 + \frac{1}{2}x^3 + \dots)(1 - 3x + 6x^2 - 10x^3 + \dots)$   
 $= 1 - 2x + 6x^2 - 10x^3 + x - 3x^2 + 6x^3 - \frac{1}{2}x^2 + \frac{1}{2}x^3 - 3x^3 + \dots$   
 $= 1 - x + 5x^2 - 10x^3 + \frac{1}{2}x^3 + \dots$   
 $= 1 - x + 5x^2 - \frac{19}{2}x^3 + \dots$

④ (i)  $(1 - x)^{\frac{1}{2}} = 1 - \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + \dots$

(ii)  $(1 - 2y + 4y^2)^{\frac{1}{2}} = (1 - 2y + 4y^2)^{\frac{1}{2}}$   
 $= (1 - 2y + 4y^2)^{\frac{1}{2}} = 1 - y + \frac{1}{2}y^2 + \dots$

⑤ (i)  $(1 + x)^{\frac{1}{3}} = 1 + \frac{1}{3}x - \frac{2}{9}x^2 + \frac{14}{27}x^3 + \dots$

(ii) (a)  $(8 + 16x)^{\frac{1}{3}} = 8^{\frac{1}{3}} (1 + 2x)^{\frac{1}{3}} = 2 (1 + 2x)^{\frac{1}{3}}$   
 $= 2 [1 + \frac{1}{3}(2x) - \frac{2}{9}(2x)^2 + \frac{14}{27}(2x)^3 + \dots]$   
 $= 2 [1 + \frac{2}{3}x - \frac{8}{9}x^2 + \frac{28}{27}x^3 + \dots]$   
 $= 2 + \frac{4}{3}x - \frac{16}{9}x^2 + \frac{56}{27}x^3 + \dots$

(b)  $1 + 2x < 1$  (iii)  
 $2x < 0$   
 $x < 0$