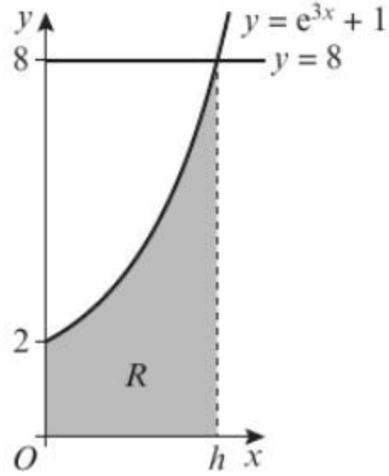
The diagram shows part of the curve $y = e^{3x} + 1$ and the line $y = 8$.

The curve and the line intersect at the point $(h, 8)$.

a Find h, giving your answer in terms of natural logarithms. **(3 marks)**

The region R is bounded by the curve, the x-axis, the y-axis and the line $x = h$.

b Use integration to show the area of R is $2 + \frac{1}{3}\ln 7$. **(5 marks)**



a When θ is small, show that the equation

$$32\cos 5\theta + 203\tan 10\theta = 182$$

can be written as

$$40\theta^2 - 203\theta + 15 = 0$$

b Hence, find the solutions of the equation

$$32\cos 5\theta + 203\tan 10\theta = 182$$

c Comment on the validity of your solutions.

a Given that α is acute and $\tan\alpha = \frac{3}{4}$, prove that

$$3\sin(\theta + \alpha) + 4\cos(\theta + \alpha) \equiv 5\cos\theta$$

b Given that $\sin x = 0.6$ and $\cos x = -0.8$, evaluate $\cos(x + 270°)$ and $\cos(x + 540°)$.

Using known trigonometric identities, prove the following:

a $\sec\theta\,\text{cosec}\,\theta \equiv 2\,\text{cosec}\,2\theta$

b $\tan\left(\frac{\pi}{4} + x\right) - \tan\left(\frac{\pi}{4} - x\right) \equiv 2\tan 2x$

c $\sin(x + y)\sin(x - y) \equiv \cos^2 y - \cos^2 x$

d $1 + 2\cos 2\theta + \cos 4\theta \equiv 4\cos^2\theta\cos 2\theta$

$$f(x) = \frac{6 + 3x - x^2}{x^3 + 2x^2},\ x > 0$$

a Express f(x) in partial fractions. **(4 marks)**

b Hence find the exact value of $\int_2^4 \frac{6 + 3x - x^2}{x^3 + 2x^2}\,dx$, writing your answer in the form $a + \ln b$, where a and b are rational numbers to be found. **(5 marks)**

$$\frac{32x^2 + 4}{(4x + 1)(4x - 1)} \equiv A + \frac{B}{4x + 1} + \frac{C}{4x - 1}$$

a Find the value of the constants A, B and C. **(4 marks)**

b Hence find the exact value of $\int_1^2 \frac{32x^2 + 4}{(4x + 1)(4x - 1)}\,dx$ writing your answer in the form $2 + k\ln m$, giving the values of the rational constants k and m. **(5 marks)**

a $\frac{1}{3}\ln 7$

b $\int_0^{\frac{1}{3}\ln 7}(e^{3x}+1)\,dx = \left[\frac{e^{3x}}{3}+x\right]_0^{\frac{1}{3}\ln 7}$

$= \left(\frac{7}{3}+\frac{1}{3}\ln 7\right) - \left(\frac{1}{3}-0\right) = 2 + \frac{1}{3}\ln 7$

a $32\cos 5\theta + 203\tan 10\theta = 182$

$32\left(1-\frac{(5\theta)^2}{2}\right) + 203\,(10\theta) = 182$

$32 - 16(25\theta^2) + 2030\theta = 182$

$0 = 400\theta^2 - 2030\theta + 150$

$0 = 40\theta^2 - 203\theta + 15$

b $5, \frac{3}{40}$

c 5 is not valid as it is not "small". $\frac{3}{40}$ is "small" so is valid.

a Find $\sin a = \frac{3}{5}$ and $\cos \alpha = \frac{4}{5}$ and insert in expansions on L.H.S. Result follows.

b 0.6, 0.8

a $f(x) = \frac{3}{x^2} - \frac{1}{x+2}$ **b** $a = \frac{3}{4}, b = \frac{2}{3}$

a $f(x) = 2 - \frac{3}{4x+1} + \frac{3}{4x-1}$, so $A = 2, B = -3$ and $C = 3$

b $k = \frac{3}{4}, m = \frac{35}{27}$