

1 When  $\theta$  is small, find the approximate values of:

a  $\frac{\sin 4\theta - \tan 2\theta}{3\theta}$

b  $\frac{1 - \cos 2\theta}{\tan 2\theta \sin \theta}$

c  $\frac{3 \tan \theta - \theta}{\sin 2\theta}$

2 When  $\theta$  is small, show that:

a  $\frac{\sin 3\theta}{\theta \sin 4\theta} = \frac{3}{4\theta}$

b  $\frac{\cos \theta - 1}{\tan 2\theta} = -\frac{\theta}{4}$

c  $\frac{\tan 4\theta + \theta^2}{3\theta - \sin 2\theta} = 4 + \theta$

3 a Find  $\cos(0.244 \text{ rad})$  correct to 6 decimal places.

b Use the approximation for  $\cos \theta$  to find an approximate value for  $\cos(0.244 \text{ rad})$ .

c Calculate the percentage error in your approximation.

d Calculate the percentage error in the approximation for  $\cos 0.75 \text{ rad}$ .

e Explain the difference between your answers to parts c and d.

4 The percentage error for  $\sin \theta$  for a given value of  $\theta$  is 1%. Show that  $100\theta = 101 \sin \theta$ .

5 a When  $\theta$  is small, show that the expression  $\frac{4 \cos 3\theta - 2 + 5 \sin \theta}{1 - \sin 2\theta}$  can be written as  $9\theta + 2$ . **(3 marks)**

b Hence write down the value of  $\frac{4 \cos 3\theta - 2 + 5 \sin \theta}{1 - \sin 2\theta}$  when  $\theta$  is small. **(1 mark)**

1 a  $\frac{2}{3}$       b 1      c 1

2 a  $\frac{\sin 3\theta}{\theta \sin 4\theta} \approx \frac{3\theta}{\theta \times 4\theta} = \frac{3\theta}{4\theta^2} = \frac{3}{4\theta}$

b  $\frac{\cos \theta - 1}{\tan 2\theta} \approx \frac{1 - \frac{\theta^2}{2} - 1}{2\theta} = \frac{-\frac{\theta^2}{2}}{2\theta} = -\frac{\theta}{4}$

c  $\frac{\tan 4\theta + \theta^2}{3\theta - \sin 2\theta} \approx \frac{4\theta + \theta^2}{3\theta - 2\theta} = \frac{4\theta + \theta^2}{\theta} = 4 + \theta$

3 a 0.970379      b 0.970232

c -0.015%      d -1.77%

e The larger the value of  $\theta$  the less accurate the approximation is.

4  $\frac{\theta - \sin \theta}{\sin \theta} \times 100 = 1 \Rightarrow (\theta - \sin \theta) \times 100 = \sin \theta$   
 $\Rightarrow 100\theta - 100 \sin \theta = \sin \theta \Rightarrow 100\theta = 101 \sin \theta.$

5 a  $\frac{4 \cos 3\theta - 2 + 5 \sin \theta}{1 - \sin 2\theta} \approx \frac{4\left(1 - \frac{(3\theta)^2}{2}\right) - 2 + 5\theta}{1 - 2\theta}$

$$= \frac{4\left(1 - \frac{9\theta^2}{2}\right) - 2 + 5\theta}{1 - 2\theta} = \frac{4 - 18\theta^2 - 2 + 5\theta}{1 - 2\theta}$$

$$= \frac{(1 - 2\theta)(9\theta + 2)}{1 - 2\theta} = 9\theta + 2$$

b 2