Trickier Projectiles – Equations of Motion

(1) A particle is launched from a point on a horizontal plane with initial velocity $U\,\text{m s}^{-1}$ at an angle of elevation α. The particle moves freely under gravity until it strikes the plane.
The greatest height of the particle is h m.

Show that $h = \dfrac{U^2 \sin^2 \alpha}{2g}$

(2) A particle is projected from a point with speed $21\,\text{m s}^{-1}$ at an angle of elevation α and moves freely under gravity. When the particle has moved a horizontal distance x m, its height above the point of projection is y m.

a Show that $y = x \tan \alpha - \dfrac{x^2}{90 \cos^2 \alpha}$

b Given that $y = 8.1$ when $x = 36$, find the value of $\tan \alpha$.

(3) A projectile is launched from a point on a horizontal plane with initial speed $U\,\text{m s}^{-1}$ at an angle of elevation α. The particle moves freely under gravity until it strikes the plane. The range of the projectile is R m.

a Show that the time of flight of the particle is $\dfrac{2U \sin \alpha}{g}$ seconds.

b Show that $R = \dfrac{U^2 \sin 2\alpha}{g}$. * Use the fact that $\sin 2\alpha = 2\sin\alpha\cos\alpha$

c Deduce that, for a fixed u, the greatest possible range is when $\alpha = 45°$.

d Given that $R = \dfrac{2U^2}{5g}$, find the two possible values of the angle of elevation at which the projectile could have been launched.

(4) A particle is projected from a point with speed U at an angle of elevation α above the horizontal and moves freely under gravity. When it has moved a horizontal distance x, its height above the point of projection is y.

a Show that $y = x \tan \alpha - \dfrac{gx^2}{2U^2}(1 + \tan^2 \alpha)$ **(5 marks)**

An athlete throws a javelin from a point P at a height of 2 m above horizontal ground. The javelin is projected at an angle of elevation of 45° with a speed of $30\,\text{m s}^{-1}$. By modelling the javelin as a particle moving freely under gravity,

b find, to 3 significant figures, the horizontal distance of the javelin from P when it hits the ground **(5 marks)**

c find, to 2 significant figures, the time elapsed from the point the javelin is thrown to the point it hits the ground. **(2 marks)**

(5) In this question **i** and **j** are unit vectors in a horizontal and upward vertical direction respectively. An object is projected from a fixed point A on horizontal ground with velocity $(k\mathbf{i} + 2k\mathbf{j})\,\text{m s}^{-1}$, where k is a positive constant. The object moves freely under gravity until it strikes the ground at B, where it immediately comes to rest. Relative to O, the position vector of a point on the path of the object is $(x\mathbf{i} + y\mathbf{j})$ m.

a Show that $y = 2x - \dfrac{gx^2}{2k^2}$ **(5 marks)**

Given that $AB = R$ m and the maximum vertical height of the object above the ground is H m,

b using the result in part **a**, or otherwise, find, in terms of k and g,

i R **ii** H **(6 marks)**