

| Question | Scheme | | Marks |
|----------|---|---|----------------|
| 1 | $b^2 - 4ac < 0$ $(3k + 1)^2 + 4k < 0$ | Correct use of discriminant. | M1 |
| | $9k^2 + 10k + 1 < 0$ | Forms quadratic inequality. | A1 |
| | $(9k + 1)(k + 1) < 0$ or $k = \frac{-10 \pm \sqrt{10^2 - 4 \times 9 \times 1}}{2 \times 9}$ | Attempts to find critical values by solving the quadratic. | M1 |
| | $k = -1, -\frac{1}{9}$ | | A1 |
| | $-1 < k < -\frac{1}{9}$ | | A1 |
| | | | 5 marks |
| 2 (a) | $(x - 2)^2 + (y + 1)^2 = 20$ | Rearranges equation and attempts to complete the square for terms in either x or y . | M1 |
| | $C(2, -1)$ | Correct coordinates given. | A1 |
| | $r = \sqrt{20}$ (or $2\sqrt{5}$) | Exact value of radius seen. Ignore subsequent rounding. | A1 |
| | | | 3 |
| (b) | $4^2 + (-5)^2 - 4 \times 4 + 2 \times (-5) = 15$ [∴ P lies on the circle.] | Correct substitution of $x = 4$ and $y = -5$. | A1 |
| | | | 1 |
| (c) | Gradient of radius = $\frac{-4}{2} = -2$ | Correct value seen. | B1 |
| | Gradient of tangent = $\frac{1}{2}$ | Finds negative reciprocal of <i>their</i> radius gradient. | M1 |
| | $y + 5 = \frac{1}{2}(x - 4)$ or $-5 = \frac{1}{2} \times 4 = c$ | Substitutes <i>their</i> tangent gradient and $x = 4$ and $y = -5$ into either $(y - y_1) = m(x - x_1)$ or $y = mx + c$. This M1 dependent on previous M1. | dM1 |
| | $y = \frac{1}{2}x - 7$ or $x - 2y - 14 = 0$ | A correct equation seen. Do not penalise subsequent incorrect versions. | A1 |
| | | | 4 |
| | | | 8 marks |
| 3 (a) | $2x = 240^\circ, 300^\circ, 600^\circ, 660^\circ$ | Evidence of finding values of $2x$ in range $0^\circ \leq x \leq 720^\circ$ (e.g. sketch of sine curve, CAST) | M1 |
| | $x = 120^\circ, 150^\circ, 300^\circ, 330^\circ$ | First A1 for two correct values. Second A1 for all four values. | A1 A1 |
| | | | 3 |
| (b) | $2(1 - \sin^2 \theta) + \sin \theta = 1$ | Attempts to use $\sin^2 \theta + \cos^2 \theta = 1$ | M1 |

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| | | to form quadratic equation in $\sin x$. | |
| | $2 \sin^2 \theta - \sin \theta - 1 = 0$ | Correct quadratic equation. | A1 |
| | $\sin \theta = -\frac{1}{2}, 1$ | Finds two roots of quadratic equation. | A1 |
| | $\theta = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$ or $\theta = 1.57, 3.67, 5.76$ | First A1 for any correct value of θ . Second A1 for three correct values. (Accept decimal equivalents and ignore any values outside the range). | A1 A1 |
| | | | 5 |
| | | | 8 marks |
| 4 | $f(1) = 1^3 + a + b + 8 = 0$ $f(2) = 2^3 + 4a + 2b + 8 = 0$ | Evidence of use of factor theorem. | M1 |
| | $a + b = -9$ $2a + b = -8$ | Simplifies to form at least one correct equation. | A1 |
| | | Attempts to solve <i>their</i> simultaneous equations | M1 |
| | $a = 1, b = -10$ | Correct values. | A1 |
| | | | 4 marks |
| | ALTERNATIVE METHOD | | |
| | $f(x) = (x^2 - 3x + 2)(x + 4)$ | Expresses correctly as product of quadratic and linear function. | M1 |
| | $f(x) = x^3 + x^2 - 10x + 8$ | Expands to form correct cubic equation. | A1 |
| | | Equates coefficients. | M1 |
| | $a = 1, b = -10$ | Correct values. | A1 |
| | | | 4 marks |
| 5(a) | $y = (x + 1)^2 - 2(x + 1) + 4$ | Evidence of use of $g(x + 1)$. | M1 |
| | $y = x^2 + 3$ | Expands and simplifies to form correct equation. | A1 |
| | | | 2 |
| (b) | $y = 2(x^2 - 2x + 4)$ | Evidence of use of $2g(x)$ (may be implied by correct answer). | M1 |
| | $y = 2x^2 - 4x + 8$ | | A1 |
| | | | 2 |
| | | | 4 marks |
| 6 (a) | $A = 2\pi rh + 2\pi r^2$ $(V =)\pi r^2 h = 500$ | Forms correct equations for A and V . | B1 |
| | $h = \frac{500}{r^2}$ | Uses a correct method to eliminate h from surface area equation. | M1 |
| | $A = 2\pi r^2 + \frac{1000}{r^2}$ | Convincing algebraic manipulation. | A1 |
| | | | 3 |
| (b) | $\frac{dA}{dr} = 4\pi r - \frac{1000}{r^2}$ | Differentiates correctly. | B1 |
| | Minimum when $\frac{dA}{dr} = 0$ $r^3 = \frac{1000}{4\pi}$ | Puts their $\frac{dA}{dr} = 0$ and attempts to solve. | M1 |
| | $r = 4.30$ | Correct value. | A1 |
| | | | 3 |

| | | | |
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| (c) | $A = 2\pi(\text{their } r)^2 + \frac{1000}{(\text{their } r)}$ | | M1 |
| | $A = 349 \text{ cm}^2$ | Ignore units. | A1 |
| | | | 2 |
| | | | 8 marks |
| 7 | $y = \int 6x^2 - 2x + 3 \, dx$ | Evidence of use of integration. As seen here or at least 2 correctly integrated terms. | M1 |
| | $(y =) 2x^3 - x^2 + 3x(+c)$ | Correct integral. | A1 |
| | $c = -2$ | Substitutes $x = 1, y = 2$ to find c . Both x and y seen to be substituted, or no more than one error out of the five terms. | M1 |
| | $y = 2x^3 - x^2 + 3x - 2$ | ' $y =$ ' now required. | A1 |
| | | | 4 marks |
| 8 (a) | $\log x - \log y^3 + \log z^4$ | Use of power rule for logs. | M1 |
| | $\log \frac{xz^4}{y^3}$ | cao | A1 |
| | | | 2 |
| (b) | $\ln \frac{2x+1}{x} = 2$ | Forms single logarithm. | M1 |
| | $\frac{2x+1}{x} = e^2$ | e^2 seen in a correct context | M1 |
| | $x = \frac{1}{e^2 - 2}$ | Correct rearrangement. | A1 |
| | | | 3 |
| | | | 5 marks |
| 9 (a) | $y < 1$ or $f(x) < 1$ | | B1 |
| | | | 1 |
| (b) | $x = \frac{-1}{y-1}$ | Evidence of rearrangement and intention to change the subject | M1 |
| | $f^{-1}(x) = \frac{-1}{x-1}$ | Do not allow y or $f(x) = \dots$ | A1 |
| | Domain: $x < 1$ | Follow through from their (a) | B1 |
| | Range: $y > 0$ or $f^{-1}(x) > 0$ | If x and y wrong way round, penalise 1 mark | B1 |
| | | | 4 |
| | | | 5 marks |
| 10 | $\frac{dy}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$ | Attempts to use product rule. Given for any form of the product rule written, or an attempt to substitute into the formula if du/dx and dv/dx seen, or an attempt to substitute into the formula where at least one of the derivatives is correct. | M1 |
| | $u = e^{2x}$ $(\frac{du}{dx} =) 2e^{2x}$ | Differentiates e^{2x} correctly. | B1 |
| | $v = \sin^2 x$ | Differentiates $\sin^2 x$ correctly. | B1 |

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| | $(\frac{dv}{dx} =) 2 \sin x \cos x$ | | |
| | $\frac{dy}{dx} = \sin^2 x \times 2e^{2x} + e^{2x} \times 2 \sin x \cos x$ | Substitutes <i>their</i> $u, v, \frac{du}{dx}, \frac{dv}{dx}$ correctly into product rule formula. | M1 |
| | $\frac{dy}{dx} = 2e^{2x} \sin^2 x + 2e^{2x} \sin x \cos x$ $\frac{dy}{dx} = 2e^{2x} \sin x (\sin x + \cos x)$ | Correct working leading to answer given. $\frac{dy}{dx}$ must be seen at some point in the working. | A1 |
| | | | 5 marks |
| 11 | $x^2 - x - 2 = 0$ | Equates the two functions to form this quadratic equation. | M1 |
| | $x = 2, -1$ | | A1 |
| | Area under $y = 5 + 3x - x^2$: $A = \int_{-1}^2 5 + 3x - x^2 dx$ | Attempts to find area under one of the curves using integration between <i>their</i> limits. | M1 |
| | $A = \left[5x + \frac{3x^2}{2} - \frac{x^3}{3} \right]_{-1}^2$ | Integrates one of the functions correctly. | A1 |
| | $A = \frac{33}{2}$ | | A1 |
| | Area under $y = x^2 + x + 1 dx$: $A = \int_{-1}^2 x^2 + x + 1 dx$ $A = \left[\frac{x^3}{3} + \frac{x^2}{2} + x \right]_{-1}^2$ | Integrates the other function correctly. | B1 |
| | $A = \frac{15}{2}$ | | A1 |
| | Shaded area = $\frac{33}{2} - \frac{15}{2}$ | Finds difference between <i>their</i> two areas. | M1 |
| | Shaded area = 9 | | A1 |
| | ALTERNATIVE METHOD | | |
| | $x^2 - x - 2 = 0$ | Equates the two functions to form this quadratic equation. | M1 |
| | $x = 2, -1$ | | A1 |
| | $f(x) = x^2 + x + 1 - (5 + 3x - x^2)$ or $f(x) = 5 + 3x - x^2 - (x^2 + x + 1)$ | Subtracts one equation from the other. | M1 |
| | $f(x) = 2x^2 - 2x - 4$ or $f(x) = -2x^2 + 2x + 4$ | | A1 |
| | $A = \int_{-1}^2 2 - 2x - 4 dx$ or $A = \int_{-1}^2 -2x^2 + 2x + 4 dx$ | | M1 |
| | $A = \left[\frac{2x^3}{3} - x^2 - 4x \right]_{-1}^2$ or | | A1 |

| | | | |
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| | $A = \left[-\frac{2x^3}{3} + x^2 + 4x \right]_{-1}^2$ | | |
| | $A = \left(\frac{2}{3} \times 2^3 - 2^2 - 4 \times 2 \right)$ $- \left(\frac{2}{3} \times (-1) - (-1)^2 - 4 \times (-1) \right)$ <p style="text-align: center;">or</p> $A = \left(-\frac{2}{3} \times 2^3 + 2^2 + 4 \times 2 \right)$ $- \left(-\frac{2}{3} \times (-1) + (-1)^2 + 4 \times (-1) \right)$ | | A1 |
| | $A = -9$ <p style="text-align: center;">or</p> $A = 9$ | | A1 |
| | Shaded area = 9 | | A1 |
| | | | 9 marks |
| 12 (a) | $v_x = 30 \cos 40^\circ$ $v_x = 22.98 \dots$ | Writes correct expression for horizontal or vertical component of 30 ms^{-1} . | B1 |
| | $t = \frac{34}{22.98 \dots}$ | Divides 34 by <i>their</i> v_x . | M1 |
| | $t = 1.479 \dots$ $t = 1.48$ | | A1 |
| | | | 3 |
| (b) | $v_y = 30 \sin 40^\circ - 9.8 \times (\text{their } t)$ | Correct use of $v = u + at$ to find v_y . | M1 |
| | | Uses $30 \sin 40^\circ$ when considering vertical motion. | B1 |
| | $v_y = 4.784 \dots$ | | A1 |
| | $v_y = 4.78 \text{ ms}^{-1}$ | Answer rounded correctly, and units given. | |
| | | | 3 |
| (c) | Vector triangle using <i>their</i> v_x and v_y . | Evidence of attempt to combine v_x and v_y to find speed or angle. | M1 |
| | $v = \sqrt{4.79^2 + 22.98^2}$ $v = 23.5 \text{ ms}^{-1}$ | | A1 |
| | $\theta = 11.8^\circ \text{ above the horizontal.}$ | | M1 Uses correct triangle with their v_y |
| | | | A1 |
| | | | 3 |
| | | | 10 marks |

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| 13 (a) | | | |
| | $R + T \sin 20^\circ = 50g \cos 30^\circ$ | Resolves forces perpendicular to slope – sin and cos wrong way round ok, +ve and –ve incorrect, ok. | M1 |
| | | Correct equation. | A1 |
| | $T \cos 20^\circ = 50g \sin 30^\circ + F$ | Resolves forces parallel to slope correctly. | M1 A1 |
| | $T = \frac{50g \sin 30^\circ + 0.25R}{\cos 20^\circ}$ | Uses $F = \mu R$ and attempts to solve the two simultaneous equations to find T. | M1 |
| | $T = 342 \text{ N}$ | | A1 |
| | | | 6 |
| (b) | | | M1 |
| | $T \cos 20^\circ - 50g \sin 30^\circ - F = 50 \times 5$ | Method: Correct form of equation | M1 A1 |
| | $R + T \sin 20^\circ = 50g \cos 30^\circ$ | Substitutes <i>their</i> R and attempts to rearrange to find T. | M1 |
| | $T = 586 \text{ N}$ | | A1 |
| | | | 4 |
| | | | 10 marks |
| 14 | $\bar{y} = \frac{124}{25} = 4.96$ | | B1 |
| | $\sigma_y = \sqrt{\frac{746}{25} - (\text{their } \bar{y})^2}$ | Substitutes summary statistics correctly into formula for standard deviation. | M1 |
| | $\sigma_y = 2.29$ | awrt | A1 |
| | $\bar{x} = 14.92$ | | B1 |
| | $\sigma_x = 4.58$ | awrt | A1 |
| | | | 5 marks |
| 15 | $H_0: p = 0.2$ $H_1: p < 0.2$ | Uses p to write both hypotheses. | B1 |
| | $X \sim B(40, 0.2)$ | Writes or uses $B(40, 0.2)$ | M1 |
| | $P(X \leq 3) = 0.0285$ or Critical region of $X \leq 3$ | | A1 |
| | $[0.0285 < 0.05]$ significant, reject H_0 | Do not allow conflicting statements. | M1 |
| | There is evidence to support the suppliers' claim. or The probability of a ball failing the bounce test is less than 0.2. | Writes a correct contextual conclusion (to include the words in bold). | A1 |
| | | | 5 marks |
| 16 | $P(A B) = \frac{P(A \cap B)}{P(B)}$ | Attempts to use formula for conditional probability. | M1 |
| | $P(A \cap B) = \frac{3}{7} \times \frac{7}{10} = 0.3$ | | A1 |
| | Venn diagram | Correct diagram (in a box) with 0.2, 0.3, 0.4 and 0.1. | B1 |
| | $P(A) \times P(B) \neq P(A \cap B)$ | | M1 |
| | $P(A) \times P(B) = 0.5 \times 0.7 = 0.35$ | | A1 |

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| | $0.35 \neq 0.3$ \therefore Not independent. | | |
| | ALTERNATIVE METHOD | | |
| | $P(A B) \neq P(A)$ | | M1 |
| | $\frac{3}{7} \neq 0.5$ \therefore Not independent. | | A1 |
| | | | 5 marks |
| | | | |