xercise 7D

a $51.7°, 231.7°$ b $170.1°, 350.1°$

c $56.5°, 303.5°$ d $150°, 330°$

a $\sin\left(\theta + \frac{\pi}{4}\right) \equiv \sin\theta\cos\frac{\pi}{4} + \cos\theta\sin\frac{\pi}{4}$

$$\equiv \frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta \equiv \frac{1}{\sqrt{2}}(\sin\theta + \cos\theta)$$

b $0, \frac{\pi}{2}, 2\pi$ c $0, \frac{\pi}{2}, 2\pi$

3 a $30°, 270°$ b $30°, 270°$

4 a $3(\sin x\cos y - \cos x\sin y)$
$- (\sin x\cos y + \cos x\sin y) = 0$
$\Rightarrow 2\sin x\cos y - 4\cos x\sin y = 0$
Divide throughout by $2\cos x\cos y$
$\Rightarrow \tan x - 2\tan y = 0$, so $\tan x = 2\tan y$

b Using **a** $\tan x = 2\tan y = 2\tan 45° = 2$
so $x = 63.4°, 243.4°$

5 a $0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi$ b $\pm 38.7°$

c $30°, 150°, 210°, 330°$ d $\frac{\pi}{12}, \frac{\pi}{4}, \frac{5\pi}{12}, \frac{3\pi}{4}$

e $60°, 300°, 443.6°, 636.4°$ f $\frac{\pi}{8}, \frac{5\pi}{8}$

g $\frac{\pi}{4}, \frac{5\pi}{4}$

h $0°, 30°, 150°, 180°, 210°, 330°$ i $\frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3}$

j $-104.0°, 0°, 76.0°$

k $0°, 35.3°, 144.7°, 180°, 215.3°, 324.7°, 360°$

6 $51.3°$

7 a $5\sin 2\theta = 10\sin\theta\cos\theta$, so equation becomes
$10\sin\theta\cos\theta + 4\sin\theta = 0$, or $2\sin\theta(5\cos\theta + 2) = 0$

b $0°, 180°, 113.6°, 246.4°$

8 a $2\sin\theta\cos\theta + \cos^2\theta - \sin^2\theta = 1$
$\Rightarrow 2\sin\theta\cos\theta - 2\sin^2\theta = 0$
$\Rightarrow 2\sin\theta(\cos\theta - \sin\theta) = 0$

b $0°, 180°, 45°, 225°$

9 a L.H.S. $= \cos^2 2\theta + \sin^2 2\theta - 2\sin 2\theta\cos 2\theta$
$= 1 - \sin 4\theta =$ R.H.S.

b $\frac{\pi}{24}, \frac{17\pi}{24}$

Exercise 7D

(P) 1 Solve, in the interval $0 \leqslant \theta < 360°$, the following equations. Give your answers to 1 d.p.

a $3\cos\theta = 2\sin(\theta + 60°)$

b $\sin(\theta + 30°) + 2\sin\theta = 0$

c $\cos(\theta + 25°) + \sin(\theta + 65°) = 1$

d $\cos\theta = \cos(\theta + 60°)$

(E/P) 2 a Show that $\sin\left(\theta + \frac{\pi}{4}\right) \equiv \frac{1}{\sqrt{2}}(\sin\theta + \cos\theta)$ **(2 marks)**

b Hence, or otherwise, solve the equation $\frac{1}{\sqrt{2}}(\sin\theta + \cos\theta) = \frac{1}{\sqrt{2}}$, $0 \leqslant \theta \leqslant 2\pi$. **(4 marks)**

c Use your answer to part **b** to write down the solutions to $\sin\theta + \cos\theta = 1$ over the same interval. **(2 marks)**

(P) 3 a Solve the equation $\cos\theta\cos 30° - \sin\theta\sin 30° = 0.5$, for $0 \leqslant \theta \leqslant 360°$.

b Hence write down, in the same interval, the solutions of $\sqrt{3}\cos\theta - \sin\theta = 1$.

(P) 4 a Given that $3\sin(x - y) - \sin(x + y) = 0$, show that $\tan x = 2\tan y$.

b Solve $3\sin(x - 45°) - \sin(x + 45°) = 0$, for $0 \leqslant x \leqslant 360°$.

(P) 5 Solve the following equations, in the intervals given.

a $\sin 2\theta = \sin\theta$, $0 \leqslant \theta \leqslant 2\pi$

b $\cos 2\theta = 1 - \cos\theta$, $-180° < \theta \leqslant 180°$

c $3\cos 2\theta = 2\cos^2\theta$, $0 \leqslant \theta < 360°$

d $\sin 4\theta = \cos 2\theta$, $0 \leqslant \theta \leqslant \pi$

e $3\cos\theta - \sin\frac{\theta}{2} - 1 = 0$, $0 \leqslant \theta < 720°$

f $\cos^2\theta - \sin 2\theta = \sin^2\theta$, $0 \leqslant \theta \leqslant \pi$

g $2\sin\theta = \sec\theta$, $0 \leqslant \theta \leqslant 2\pi$

h $2\sin 2\theta = 3\tan\theta$, $0 \leqslant \theta < 360°$

i $2\tan\theta = \sqrt{3}(1 - \tan\theta)(1 + \tan\theta)$, $0 \leqslant \theta \leqslant 2\pi$

j $\sin^2\theta = 2\sin 2\theta$, $-180° < \theta < 180°$

k $4\tan\theta = \tan 2\theta$, $0 \leqslant \theta \leqslant 360°$

(E/P) 6 In $\triangle ABC$, $AB = 4$ cm, $AC = 5$ cm, $\angle ABC = 2\theta$ and $\angle ACB = \theta$. Find the value of θ, giving your answer, in degrees, to 1 decimal place. **(4 marks)**

(E/P) 7 a Show that $5\sin 2\theta + 4\sin\theta = 0$ can be written in the form $a\sin\theta(b\cos\theta + c) = 0$, stating the values of a, b and c. **(2 marks)**

b Hence solve, for $0 \leqslant \theta < 360°$, the equation $5\sin 2\theta + 4\sin\theta = 0$. **(4 marks)**

(E/P) 8 a Given that $\sin 2\theta + \cos 2\theta = 1$, show that $2\sin\theta(\cos\theta - \sin\theta) = 0$. **(2 marks)**

b Hence, or otherwise, solve the equation $\sin 2\theta + \cos 2\theta = 1$ for $0 \leqslant \theta < 360°$. **(4 marks)**

(E/P) 9 a Prove that $(\cos 2\theta - \sin 2\theta)^2 \equiv 1 - \sin 4\theta$. **(4 marks)**

b Use the result to solve, for $0 \leqslant \theta < \pi$, the equation $\cos 2\theta - \sin 2\theta = \frac{1}{\sqrt{2}}$

Give your answers in terms of π. **(3 marks)**

$B = 1 + \frac{\sqrt{3}}{4}$

$E = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$E = \frac{3+4\sqrt{3}}{10}$

$Y = \frac{30\sqrt{3}+40}{11}$

$R = \frac{4+3\sqrt{3}}{10}$

$L = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$E = 2-\sqrt{3}$

$Q = {}^{2}/_{5}$

$U = -1$

$K = -{}^{3}/_{5}$

$T = \frac{\sqrt{3}}{2}$

$C = {}^{3}/_{5}$

$U = 8-5\sqrt{3}$

$H = 1$

$C = {}^{1}/_{7}$

$W = -{}^{1}/_{7}$

$O = {}^{4}/_{5}$

$S = 0$

$O = \frac{10(3\sqrt{3}-4)}{11}$

$M = 8+5\sqrt{3}$

$C = \frac{3-4\sqrt{3}}{10}$

$T = 3\cos\theta$

COMPOUND ANGLES

1. Without using a calculator, find the values of
   a) $\sin 75$
   b) $\cos 75$
   c) $\tan \pi/12$

2. Without using a calculator, find the values of
   a) $\dfrac{\tan 70 + \tan 65}{1 - \tan 70 \tan 65}$
   b) $\sin 80 \cos 20 - \cos 80 \sin 20$
   c) $\sin 100 \cos 10 - \cos 100 \sin 10$

3. Given that $\sin A = 4/5$ and $\sin B = 1/2$, where A and B are both acute angles, calculate the exact values of
   a) $\sin(A+B)$
   b) $\cos(A-B)$
   c) $\sec(A-B)$

4. Given that $\cos A = -4/5$ and A is an obtuse angle measured in radians, find the exact value of
   a) $\sin A$
   b) $\cos(\pi + A)$
   c) $\sin\left(\frac{\pi}{3} + A\right)$
   d) $\tan\left(\frac{\pi}{4} + A\right)$

5. Given that $\tan\left(x + \frac{\pi}{3}\right) = \frac{1}{2}$, find the exact value of $\tan x$

6. Simplify $\cos\theta + \cos\left(\theta + \frac{2\pi}{3}\right) + \cos\left(\theta + \frac{4\pi}{3}\right)$