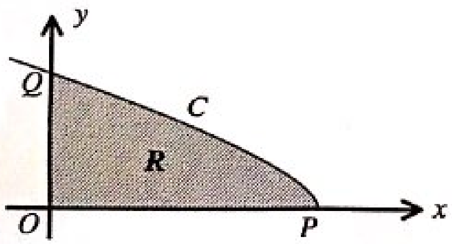
①



The figure above shows the curve C, given parametrically by

$$x = 4 - t^2, \quad y = t(t+3), \quad t \geq 0.$$

The curve meets the coordinate axes at the points P and Q.

a) Find the coordinates of P and Q.

The finite region R is bounded by C and the coordinate axes.

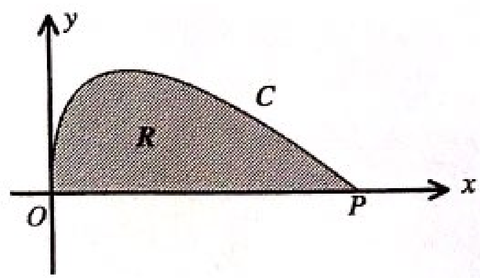
b) Show that the area of R is given by

$$\int_{t_1}^{t_2} 2t^3 + 6t^2 \, dt,$$

stating the values of t_1 and t_2.

c) Hence find the area of R.

②



The figure above shows the curve C, given parametrically by

$$x = 6t^2, \quad y = t - t^2, \quad t \geq 0.$$

The curve meets the x axis at the origin O and at the point P.

a) Find the x coordinate of P

The finite region R, bounded by C and the x axis, is revolved in the x axis by 2π radians to form a solid of revolution, whose volume is denoted by V.

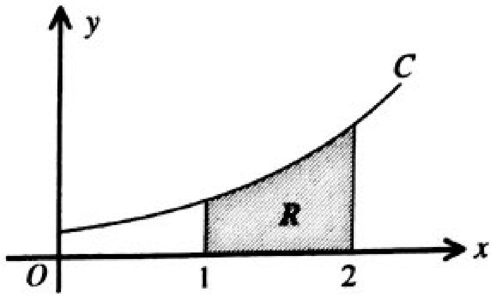
b) Show clearly that

$$V = \pi \int_0^T 12t\left(t - t^2\right)^2 \, dt,$$

stating the value of T.

c) Hence find the value of V.

③



The figure above shows the curve C, given parametrically by

$$x = \ln t, \quad y = t + \sqrt{t}, \quad 1 \le t \le 10.$$

The finite region R is bounded by C, the straight lines with equations $x = 1$ and $x = 2$, and the x axis.

a) Show that the area of R is given by

$$\int_{T_1}^{T_2} 1 + t^{-\frac{1}{2}} \, dt,$$

stating the values of T_1 and T_2.

b) Hence find an exact value for the area of R.

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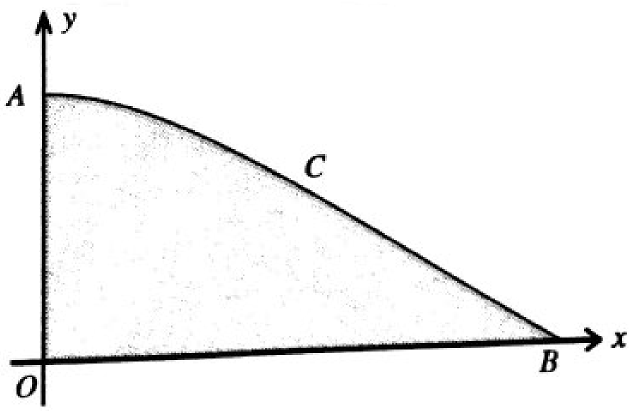
④



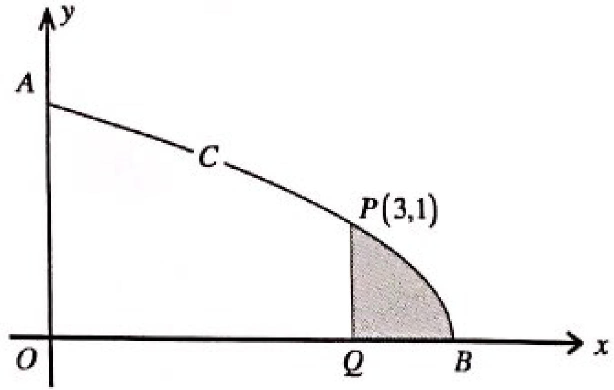
The figure above shows the curve C, with parametric equations

$$x = 36t^2 - \pi^2, \quad y = \frac{\sin 3t}{8}, \quad \frac{\pi}{6} \le t \le \frac{\pi}{3}.$$

The curve meets the coordinate axes at the points A and B.

By setting up and evaluating a suitable integral in parametric, show that the area bounded by C and the coordinate axes is $a\pi + b$ square units. State a and b

(5)



The figure above shows the curve C, with parametric equations

$$x = 4\sin^2 t\ ,\ y = 2\cos t\ ,\ 0 \le t \le \frac{\pi}{2}.$$

The curve meets the coordinate axes at the points A and B.

The point $P(3,1)$ lies on C.

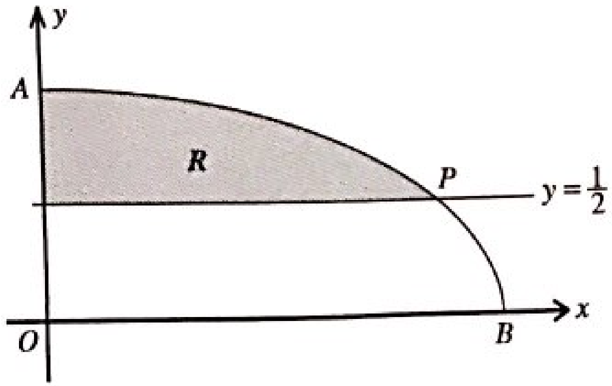
The point Q lies on the x axis so that PQ is parallel to the y axis.

a) Show that the area of the shaded region bounded by C, the line PQ and the x axis is given by the integral

$$k\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos^2 t \sin t \, dt.$$ State k

b) Evaluate the above integral to find the area of the shaded region.

(6)



The figure above shows the curve C, with parametric equations

$$x = 4\cos\theta,\quad y = \sin\theta\ ,\ 0 \le \theta \le \frac{\pi}{2}.$$

The curve meets the coordinate axes at the points A and B. The straight line with equation $y = \frac{1}{2}$ meets C at the point P.

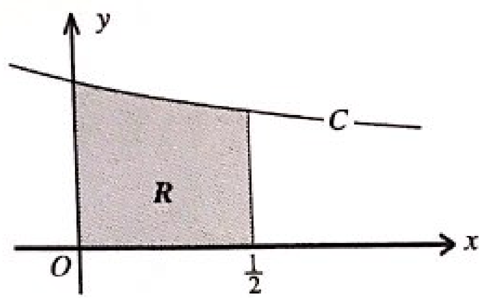
a) Show that the area under the arc of the curve between A and P, and the x axis, is given by the integral

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} p\sin^2\theta \, d\theta.$$ State p

The shaded region R is bounded by C, the straight line with equation $y = \frac{1}{2}$ and the y axis.

b) Find an exact value for the area of R.

7



The figure above shows part of the curve C, with parametric equations

$$x = \cos 2\theta, \quad y = \sec\theta, \quad 0 < \theta < \frac{\pi}{2}.$$

The finite region R is bounded by C, the straight line with equation $x = \frac{1}{2}$ and the coordinate axes.

a) Show that the area of R is given by the integral

$$\int_a^b k \sin\theta \, d\theta.$$

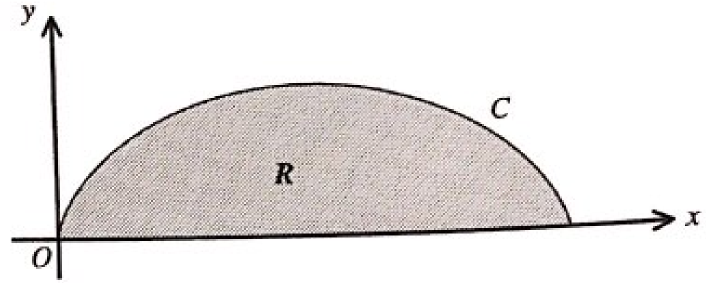
State a, b, k

b) Evaluate the above integral to find an exact value for R.

The region R is rotated by 2π radians in the x axis to form a solid of revolution S.

c) Use parametric integration to find an exact value for the volume of S.

8



The figure above shows a cycloid C, whose parametric equations are

$$x = \theta - \sin\theta, \quad y = 1 - \cos\theta, \quad 0 \le \theta \le 2\pi.$$

The finite region R is bounded by C and the x axis.

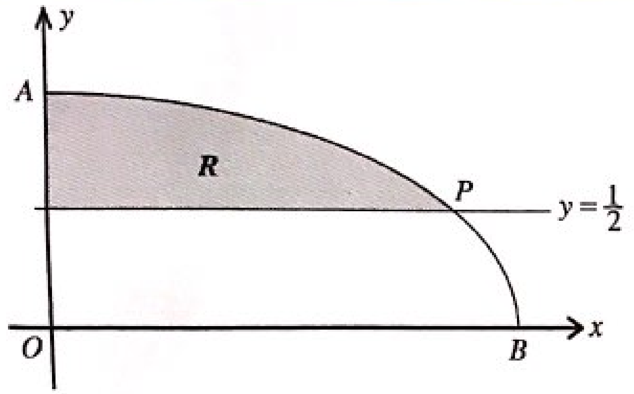
a) Show, with full justification, that the area of R is given by

$$\int_0^{2\pi} (a + b\cos\theta)^2 \, d\theta$$

State a and b

b) Hence find the area of R.

9



The figure above shows the curve C, with parametric equations

$$x = 4\cos\theta, \quad y = \sin\theta, \quad 0 \le \theta \le \frac{\pi}{2}.$$

The curve meets the coordinate axes at the points A and B. The straight line with equation $y = \frac{1}{2}$ meets C at the point P.

a) Show that the area under the arc of the curve between A and P, and the x axis, is given by the integral

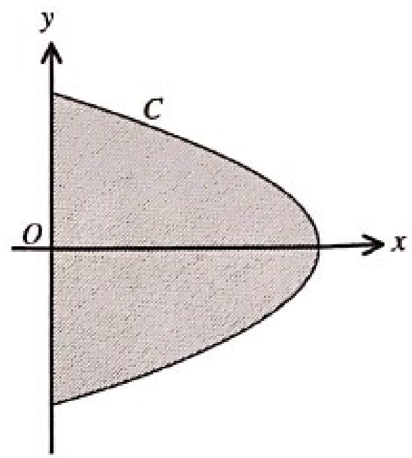
$$\int_a^b k\sin^2\theta \, d\theta.$$

State a, b, k

The shaded region R is bounded by C, the straight line with equation $y = \frac{1}{2}$ and the y axis.

b) Find an exact value for the area of R.

10



The figure above shows the curve C, given parametrically by

$$x = 5\cos^2\theta, \quad y = 6\sin\theta, \quad -\frac{\pi}{2} < \theta \le \frac{\pi}{2}.$$

The curve is symmetrical in the x axis.

The finite region bounded by C and the y axis is denoted by R.

a) Show that the area of R is given by

$$\int_a^b k\sin^2\theta\cos\theta \, d\theta.$$

State a, b, k

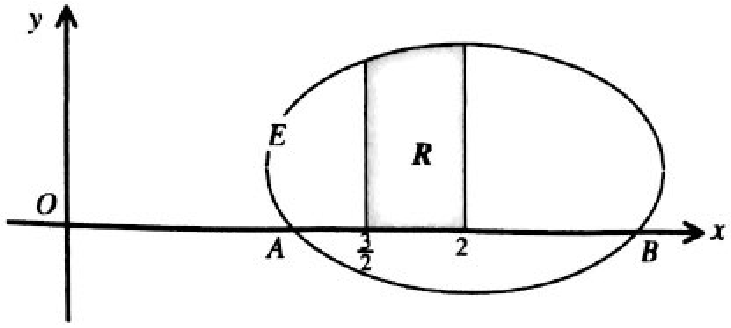
b) Hence find the area of R.

The region R is to be revolved by π radians in the x axis to form a solid of revolution S.

c) Find the exact volume* of S

* Volume = $\int_a^b \pi y^2 \, dx$

(11)



The figure above shows an ellipse E, given parametrically by

$$x = 2 - \cos\theta, \quad y = 1 + 2\sin\theta, \quad 0 \le \theta < 2\pi.$$

The ellipse crosses the x axis at the points A and B.

a) Find, as exact surds, the coordinates of A and the coordinates of B.

The finite region R is bounded by E, for which $y \ge 0$, the x axis and the straight lines with equations $x = \frac{3}{2}$ and $x = 2$.

b) Show that the area of R is given by $\int_a^b \sin\theta + k\sin^2\theta \, d\theta$ State a, b, k

c) Hence find the area of R.

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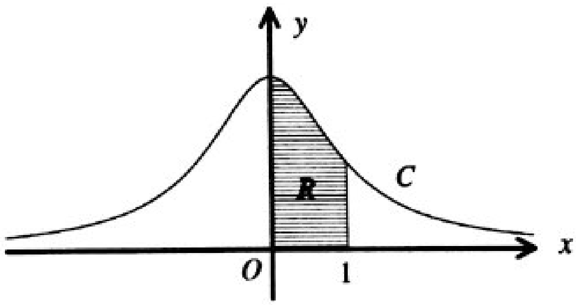
(12)



The figure above shows the curve C, defined by the parametric equations

$$x = \tan\theta, \quad y = \cos^2\theta, \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}.$$

The finite region R is bounded by C, the coordinate axes and the straight line with equation $x = 1$.

a) Determine the area of R.

The region R is revolved by 2π radians in the x axis, forming a solid S.

b) Show that the volume of S is $\frac{\pi}{a}(b\pi + c)$ State a, b, c

c) Find a Cartesian equation of C, giving the answer in the form $y = f(x)$.

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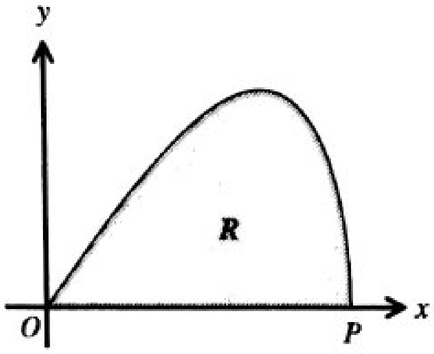
(13)



The figure above shows the curve C with parametric equations

$$x = 5\cos t, \quad y = 3\sin 2t \ , \quad 0 \le t \le \frac{\pi}{2}.$$

The curve meets the x axis at the origin O and at the point P.

a) Find the value of t at O and at P.

The finite region R bounded by C and the x axis is revolved by 2π radians in the x axis forming a solid of revolution S.

b) Show that the volume of S is given by the integral

$$a\pi \int_0^{\frac{\pi}{2}} b \ \sin^c t \cos^d t \ dt.$$

State a, b, c, d

c) By using the substitution $u = \cos t$, or otherwise, find the volume of S.

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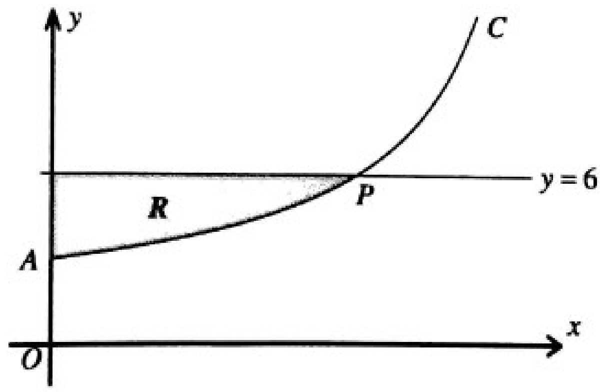
(14)



The figure above shows the curve C, with parametric equations $x = 6t\sin t$, $y = 3\sec t$, $0 \le t < \pi/2$

a) Show that the area **under** the arc of the curve between A and P, and the x axis is given by the integral $a\int_b^c pt + q\tan t \ dt$

The curve meets the coordinate axes at the point A. The line $y = 6$ meets C at the point P.

The shaded region R is bounded by C, the line $y = 6$ and the y axis.

b) Show that the area of R is approximately 10.3 square units.

① $\boxed{P(4,0)\ \&\ Q(0,10)}$, $\boxed{t_1=0}$, $\boxed{t_2=2}$, $\boxed{\text{area}=24}$

② $\boxed{6}$, $\boxed{T=1}$, $\boxed{V=\frac{\pi}{5}}$

③ $\boxed{T_1=e,\ T_2=e^2}$, $\boxed{e^2+e-2e^{\frac{1}{2}}}$,

④ $a=1,\ b=-1$

⑤ $k=16$, $\boxed{\text{area}=\frac{2}{3}}$

⑥ $p=4$, $\boxed{\text{area}=\frac{1}{6}\left(4\pi-3\sqrt{3}\right)}$

⑦ $k=4,\quad a=\frac{\pi}{4},\quad b=\frac{\pi}{6}$ $\boxed{\text{area}=2\sqrt{3}-2\sqrt{2}}$, $\boxed{\text{volume}=2\pi\ln\left(\frac{3}{2}\right)}$

⑧ $a=1,\ b=-1$ $\boxed{\text{area}=3\pi}$

⑨ $a=\frac{\pi}{6},\quad b=\frac{\pi}{2},\quad k=4$ $\boxed{\text{area}=\frac{1}{6}\left(4\pi-3\sqrt{3}\right)}$

⑩ $a=0,\quad b=\frac{\pi}{2},\quad k=120$ b) $\boxed{\text{area}=40}$ c) 90π

⑪ $\boxed{A\left(2-\frac{\sqrt{3}}{2}\right), B\left(2+\frac{\sqrt{3}}{2}\right)}$, $a=\frac{\pi}{3},\quad b=\frac{\pi}{2},\quad k=2$ $\boxed{\text{area}=\frac{1}{12}\left(6+2\pi+3\sqrt{3}\right)\approx 1.46}$

⑫ $\boxed{\text{area}=\frac{\pi}{4}}$, $a=8,\ b=1,\ c=2$ $\boxed{y=\frac{1}{1+x^2}}$

⑬ $\boxed{t_O=\frac{\pi}{2},\ t_P=0}$, $a=1,\ b=180,\ c=3,\ d=2$ $\boxed{\text{volume}=24\pi}$

⑭ $a=18,\quad b=0,\ c=\frac{\pi}{3},\ p=1,\ q=1\qquad R\approx 10.3$