Question 1 (**)

A curve is given parametrically by

$$x = 3 + 2\cos\theta, \quad y = -3 + 2\sin\theta, \quad 0 \le \theta < 2\pi .$$

Show clearly that

$$\frac{dy}{dx} = \frac{a - x}{b + y}.$$

State a,b

Question 2 (**)

A curve is defined by the following parametric equations

$$x = 4at^2, \quad y = a(2t + 1), \quad t \in \mathbb{R},$$

where a is non zero constant.

Given that the curve passes through the point $A(4,0)$, find the value of a.

Question 3 (***)

A curve C is given parametrically by the equations

$$x = 2t^2 + \frac{1}{t}, \quad y = 2t^2 - \frac{1}{t}, \quad t \in \mathbb{R}, \ t \neq 0.$$

a) Show that at the point on C where $t = \frac{1}{2}$, the gradient is m.

State m

b) By considering $(x + y)$ and $(x - y)$, show that a Cartesian equation of C is

$$(x + y)(x - y)^2 = p$$

State p

Question 4 (*+)**

The curve C_1 has Cartesian equation

$$x^2 + y^2 = 9x - 4.$$

The curve C_2 has parametric equations

$$x = t^2, \quad y = 2t, \quad t \in \mathbb{R}.$$

Find the coordinates of the points of intersection of C_1 and C_2.

Question 5 (*+)**

A curve C is defined parametrically by

$$x = t + \ln t, \quad y = t - \ln t, \quad t > 0.$$

a) Find the coordinates of the turning point of C.

b) Show that a Cartesian equation for C is

$$K e^{x-y} = (x+y)^P.$$

State K, P

Question 6 (*+)**

The curve C is given parametrically by the equations

$$x = 2e^{t} + 1, \quad y = e^{3t} - 6e^{t} + 1, \quad t \in \mathbb{R}.$$

Determine the coordinates of the point on C with $\frac{dy}{dx} = 3$.

Question 7 (****)

The curve C is given parametrically by

$$x = 1 - 3t, \quad y = \frac{t+6}{t+2}, \quad t \in \mathbb{R}.$$

a) Find a simplified expression for $\frac{dy}{dx}$, in terms of t.

b) Show that the straight line L with equation

$$4x - 3y = 1$$

is a tangent to C, and determine the coordinates of the point of tangency between L and C.

Question 8 **(****+)**

A curve C is given by the parametric equations

$$x = 2\cos 2t, \quad y = 5\sin t, \quad -\frac{\pi}{2} \le t \le \frac{\pi}{2}.$$

The point $P\left(1, \frac{5}{2}\right)$ lies on C.

a) Find the value of the gradient at P.

b) Hence, show that an equation of the normal to C at P is

$ax + by + c = 0$

state a, b, c

The normal at P meets C again at the point Q.

c) Find the y coordinate of Q .

Question 9 **(*****)**

A curve C is given parametrically by the equations

$$x = 2 + 2\sin\theta, \quad y = 2\cos\theta + \sin 2\theta, \quad 0 \le \theta < 2\pi.$$

a) By considering a simplified expression for $\frac{y}{x}$, show that a Cartesian equation of C is given by

$y^2 = ax^3 + bx^4$

state a, b

b) Given that C meets the straight line with equation $y = x$ at the origin and at the point P, determine the coordinates of P.

c) Use differentiation to show that the straight line with equation $y = x$ is in fact a tangent to C at the point P.

(1) $a=3$, $b=3$

(2) 4

(3) $m=-3$ $p=16$

(4) $\boxed{(4,4),\ (4,-4),\ (1,2),\ (1,-2)}$

(5) a) $(1,1)$ b) $k=4$, $p=2$

(6) $(5,-3)$

(7) $\boxed{\dfrac{dy}{dx}=\dfrac{4}{3(t+2)^2}}$, $\boxed{(4,5)}$

(8) a) $-5/4$ b) $a=8$, $b=-10$, $c=17$ c) $-\dfrac{165}{16}$

(9) a) $a=1$, $b=-\dfrac{1}{4}$ b) $(2,2)$