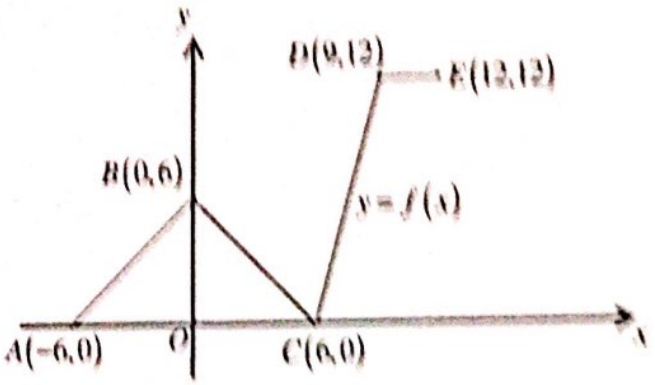
(1) The figure below shows the graph of a function with equation $y = f(x)$.

The graph consists of four straight line segments joining the points $A(-6,0)$, $B(0,6)$, $C(6,0)$, $D(9,12)$ and $E(12,12)$.



Sketch on separate diagrams the graph of ...

a) ... $y = f(x-6)$.

b) ... $y = f(-x)$.

c) ... $y = \frac{1}{2} f\left(\frac{1}{2}x\right)$.

Each sketch must include the new coordinates of A, B, C, D and E.

(2) A machine is used to produce waves in the swimming pool of a water theme park.

Let x cm be the height of the wave produced above a certain level in the pool, and suppose it can be modelled by the differential equation

$$\frac{dx}{dt} = 2x \sin 2t, \quad t > 0,$$

where t is the time in seconds.

When $t = 0$, $x = 6$.

a) Solve the differential equation to show

$$x = 6e^{1-\cos 2t}.$$

b) Find the maximum height of the wave.

(3) Relative to a fixed origin O the following position vectors are given.

$$\overrightarrow{OA} = \begin{pmatrix} 0 \\ 8 \\ 3 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 1 \\ 13 \\ 1 \end{pmatrix}.$$

a) Find a vector equation for the line straight l_1 which passes through A and B.

The straight line l_2 has vector equation

$$\mathbf{r} = \begin{pmatrix} 7 \\ 0 \\ 9 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix},$$

where μ is a scalar parameter.

b) Show that l_1 and l_2 do not intersect.

c) Find the position vector of C, given it lies on l_2 and $\measuredangle ABC = 90°$.

$\square$, $\boxed{\mathbf{r} = 8\mathbf{j} + 3\mathbf{k} + \lambda(\mathbf{i} + 5\mathbf{j} - 2\mathbf{k})}$, $\boxed{C(-3,15,4)}$

(4) A particle of mass 2 kg is dragged by a constant force of 49 N, up the line of greatest slope of a rough plane, inclined at an angle of 30° to the horizontal. This force is also acting in the line of greatest slope of the plane. The coefficient of friction between the particle and the plane is 0.5.

The particle passes through two points on the plane A and B which are 4 m apart, and point B is at a higher level on the plane than point A.

Given that the particle is passing through A with a speed of 10 ms^{-1}, use work and energy considerations to find the speed of the particle as it passes through B.

$\boxed{v \approx 14.93 \text{ ms}^{-1}}$

(5) $\displaystyle\int_0^1 \frac{1}{\left(1+x^2\right)^2}\,dx$ use $x = \tan\theta$

Answer

$\frac{1}{8}(\pi + 2)$,

$$f(x) \equiv 3\sin x + \cos x,\ x \in \mathbb{R}$$

a) Express $f(x)$ in the form $R\cos(x-\alpha)$, $R>0$, $0<\alpha<\frac{\pi}{2}$.

b) Solve the equation

$$f(x) = 2 \text{ for } 0<x<2\pi.$$

c) Write down the minimum value of $f(x)$.

d) Find the smallest positive value of x for which this minimum value occurs.

$\boxed{f(x) \equiv \sqrt{10}\cos(x-1.249^c)}$, $\boxed{x = 0.363^c, 2.135^c}$, $\boxed{f(x)_{\min} = -\sqrt{10}}$, $\boxed{x = 4.391^c}$

(5) The points $A(2,10,7)$ and $B(0,15,12)$ are given.

a) Determine a vector equation of the straight line l_1 that passes through the points A and B.

The vector equation of the straight line l_2 is

$$\mathbf{r}_2 = 4\mathbf{i} + \mathbf{j} - 6\mathbf{k} + \mu(2\mathbf{i} - \mathbf{j} + 3\mathbf{k}),$$

where μ is a scalar parameter.

b) Show that l_1 and l_2 intersect at some point P and find its coordinates.

c) Calculate the acute angle between l_1 and l_2.

$\boxed{\quad}$, $\boxed{\mathbf{r}_1 = 2\mathbf{i} + 10\mathbf{j} + 7\mathbf{k} + \lambda(-2\mathbf{i} + 5\mathbf{j} + 5\mathbf{k})}$, $\boxed{P(6,0,-3)}$, $\boxed{77.4°}$

7.

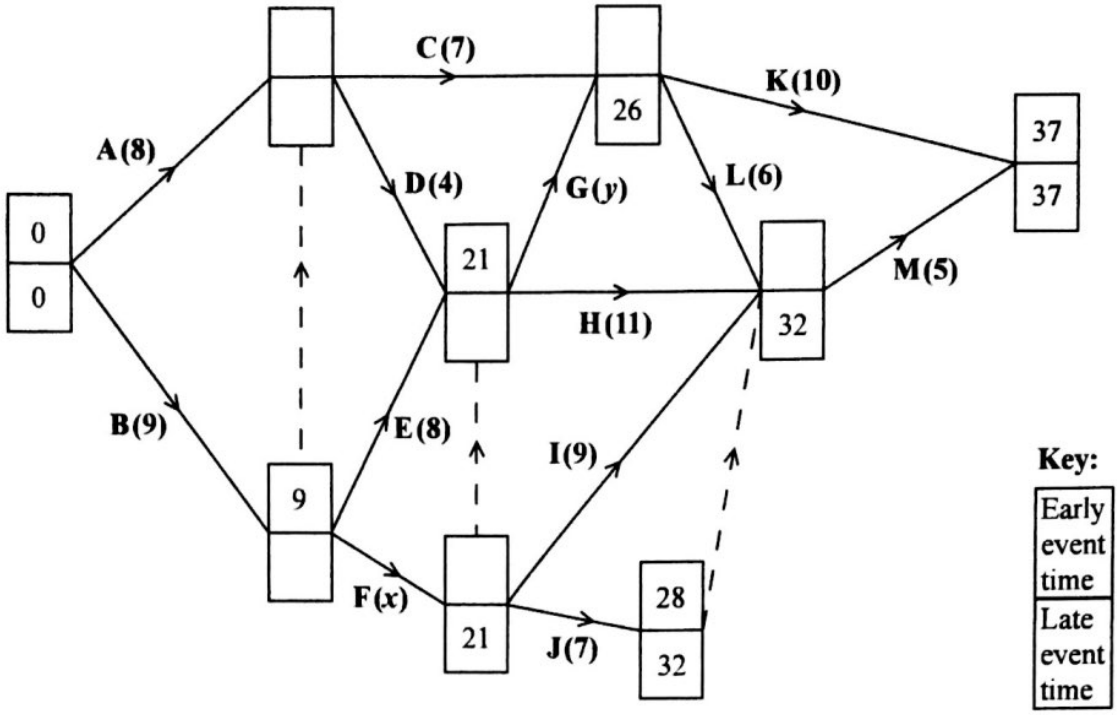


Figure 4

[*The sum of all the activity durations is 99 days*]

The network in Figure 4 shows the activities that need to be undertaken to complete a project. Each activity is represented by an arc and the duration of the activity, in days, is shown in brackets. The early event times and late event times are to be shown at each vertex and some have been completed for you.

Given that activity F is a critical activity and that the total float on activity G is 2 days,

(a) write down the value of x and the value of y,

(2)

(b) calculate the missing early event times and late event times and hence complete Diagram 1 in your answer book.

(3)

Each activity requires one worker and the project must be completed in the shortest possible time.

(c) Calculate a lower bound for the number of workers needed to complete the project in the shortest possible time.

(1)

(d) Draw a cascade (Gantt) chart for this project on Grid 1 in the answer book.

(4)

(e) Use your cascade chart to determine the minimum number of workers needed to complete the project in the shortest possible time. You must make specific reference to times and activities. (You do **not** need to provide a schedule of the activities.)

(2)

(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

END