

VECTORS

The last bits of the questions.

Always draw a diagram

1) The line l_1 has equation $\mathbf{r} = 4\mathbf{i} + 28\mathbf{j} + 4\mathbf{k} + \alpha(-\mathbf{i} - 5\mathbf{j} + \mathbf{k})$
and the line l_2 has equation $\mathbf{r} = 5\mathbf{i} + 3\mathbf{j} + \mathbf{k} + \beta(3\mathbf{i} + 4\mathbf{k})$
The lines intersect at the point X with co-ordinates (-1, 3, 9)
The angle between the lines is 74.37°
The point A lies on l_1 and has co-ordinates (2, 18, 6)

The distance AX is $9\sqrt{3}$

a) Draw a diagram to show this information

The point Y lies on l_2 . The vector YA is perpendicular to the line l_1

b) Find the distance YA

The point B lies on l_1 where $|AX| = 2|AB|$

c) Find the two possible position vectors of B

2) Relative to a fixed origin O , the point A has position vector $(2\mathbf{i} - \mathbf{j} + 5\mathbf{k})$,
the point B has position vector $(5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k})$,
and the point D has position vector $(-\mathbf{i} + \mathbf{j} + 4\mathbf{k})$.

The line l passes through the points A and B.

The vector AB is $3\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$

A vector equation for the line l is $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + 5\mathbf{k} + \beta(3\mathbf{i} + 3\mathbf{j} + 5\mathbf{k})$

The angle BAD is 109° , to the nearest degree.

a) Draw a diagram to show this information

The points A, B and D, together with a point C, are the vertices of the parallelogram ABCD,
where $AB = DC$

b) Find the position vector of C.

c) Find the area of the parallelogram ABCD, giving your answer to 3 significant figures.

d) Find the shortest distance from the point D to the line l , giving your answer to 3 significant figures.

3) With respect to a fixed origin O , the lines l_1 and l_2 are given by the equations
 $\mathbf{r} = 6\mathbf{i} - 3\mathbf{j} - 2\mathbf{k} + \alpha(-\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$ and $\mathbf{r} = -5\mathbf{i} + 15\mathbf{j} + 3\mathbf{k} + \alpha(2\mathbf{i} - 3\mathbf{j} + \mathbf{k})$

l_1 and l_2 meet at the point (3, 3, 7)

The acute angle between l_1 and l_2 is 69.1°

The point B has co-ordinates (5, -1, 1) and lies on l_1

a) Draw a diagram to show this information

b) Find the shortest distance from B to the line l_2

4) With respect to a fixed origin O , the lines l_1 and l_2 are given by the equations
 $\mathbf{r} = 11\mathbf{i} + 2\mathbf{j} + 17\mathbf{k} + \alpha(-2\mathbf{i} + \mathbf{j} - 4\mathbf{k})$ and $\mathbf{r} = -5\mathbf{i} + 11\mathbf{j} + \mathbf{k} + \alpha(-3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$

The lines l_1 and l_2 intersect at the point (1, 7, -3)

l_1 and l_2 intersect at right angles

The point A lies on l_1 and has co-ordinates (9, 3, 13)

The point C lies on l_2

A circle with centre C cuts the line l_1 at the points A and B

a) Draw a diagram to show this information

b) Find the co-ordinates of B