1. The line l_1 has vector equation $\mathbf{r} = 8\mathbf{i} + 12\mathbf{j} + 14\mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} - \mathbf{k})$, where λ is a parameter.

The point A has coordinates $(4, 8, a)$, where a is a constant. The point B has coordinates $(b, 13, 13)$, where b is a constant. Points A and B lie on the line l_1

(*a*) Find the values of a and b. **(3)**

Given that the point O is the origin, and that the point P lies on l_1 such that OP is perpendicular to l_1,

(*b*) find the coordinates of P. **(5)**

(*c*) Hence find the distance OP, giving your answer as a simplified surd. **(2)**

2. The line l_1 has vector equation $\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$

and the line l_2 has vector equation $\mathbf{r} = \begin{pmatrix} 0 \\ 4 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$,

where λ and μ are parameters.

The lines l_1 and l_2 intersect at the point B and the acute angle between l_1 and l_2 is θ.

(*a*) Find the coordinates of B. **(4)**

(*b*) Find the value of $\cos \theta$, giving your answer as a simplified fraction. **(4)**

The point A, which lies on l_1, has position vector $\mathbf{a} = 3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$.

The point C, which lies on l_2, has position vector $\mathbf{c} = 5\mathbf{i} - \mathbf{j} - 2\mathbf{k}$.

The point D is such that $ABCD$ is a parallelogram.

(*c*) Show that $|\overrightarrow{AB}| = |\overrightarrow{BC}|$. **(3)**

(*d*) Find the position vector of the point D. **(2)**

3. The point A, with coordinates $(0, a, b)$ lies on the line l_1, which has equation

$$\mathbf{r} = 6\mathbf{i} + 19\mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}).$$

(*a*) Find the values of a and b. **(3)**

The point P lies on l_1 and is such that OP is perpendicular to l_1, where O is the origin.

(*b*) Find the position vector of point P. **(6)**

Given that B has coordinates $(5, 15, 1)$,

(*c*) show that the points A, P and B are collinear and find the ratio $AP : PB$. **(4)**

4. The point A has position vector $\mathbf{a} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and the point B has position vector $\mathbf{b} = \mathbf{i} + \mathbf{j} - 4\mathbf{k}$, relative to an origin O.

(*a*) Find the position vector of the point C, with position vector $\mathbf{c}$, given by $\mathbf{c} = \mathbf{a} + \mathbf{b}$. **(1)**

(*b*) Show that $OACB$ is a rectangle, and find its exact area. **(6)**

The diagonals of the rectangle, AB and OC, meet at the point D.

(*c*) Write down the position vector of the point D. **(1)**

(*d*) Find the size of the angle ADC. **(6)**

5. The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$.

The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 3 \\ 6 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$.

(*a*) Show that l_1 and l_2 do not meet. **(4)**

The point A is on l_1 where $\lambda = 1$, and the point B is on l_2 where $\mu = 2$.

(*b*) Find the cosine of the acute angle between AB and l_1. **(6)**

6. Relative to a fixed origin O, the point A has position vector $(10\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$, and the point B has position vector $(8\mathbf{i} + 3\mathbf{j} + 4\mathbf{k})$.

The line l passes through the points A and B.

(*a*) Find the vector $\overrightarrow{AB}$. **(2)**

(*b*) Find a vector equation for the line l. **(2)**

The point C has position vector $(3\mathbf{i} + 12\mathbf{j} + 3\mathbf{k})$.

The point P lies on l. Given that the vector $\overrightarrow{CP}$ is perpendicular to l,

(*c*) find the position vector of the point P. **(6)**

1. a) a=18, b= 9 b) (6,10,16) c) $14\sqrt{2}$

2. a) (2,2,-2) b) $\frac{1}{3}$ d) 6**i**-2**j**+2**k**

3. a) a=-5, b=11 b) 2**i**+3**j**+7**k** c) 2:3

4. a) 3**i** + 3**j** – 3**k** b) $9\sqrt{2}$ c) $\frac{3}{2}$**i** + $\frac{3}{2}$**j** – $\frac{3}{2}$**k** d) 109°

5. b) $\frac{7}{10}$

6. a) -2**i**+**j**+**k** b) r = 10**i** + 2**j** + 3**k** + t(-2**i** + **j** + **k**) c) 2**i** + 6**j** + 7**k**