SOCRATIVE

(1) An oil spillage on the surface of the sea remains circular at all times.

The radius of the spillage, r km, is increasing at the constant rate of $0.5\ \text{km h}^{-1}$.

a) Find the rate at which the area of the spillage, $A\ \text{km}^2$, is increasing, when the circle's radius has reached 10 km.

A different oil spillage on the surface of the sea also remains circular at all times.

The area of this spillage, $A\ \text{km}^2$, is increasing at the rate of $0.5\ \text{km}^2\text{h}^{-1}$.

b) Show that when the area of the spillage has reached $10\ \text{km}^2$, the rate at which the radius r of the spillage is increasing is

$$\frac{1}{4\sqrt{10\pi}}\ \text{km h}^{-1}.$$

(2) The variables y, x and t are related by the equations

$$y = 10e^{\frac{1}{3}x-1} \quad \text{and} \quad x = \sqrt{6t+1},\ t \geq 0.$$

Find the value of $\frac{dy}{dt}$, when $t = 4$.

(3) An extended ladder AB, of length 20 m, has one end A on level horizontal ground and the other end B resting against a vertical wall.

The end A begins to slip away from the wall with constant speed $0.3\ \text{ms}^{-1}$, and the end B slips down the wall.

Determine the speed of the end B, when B has reached a height of 12 m above the ground.

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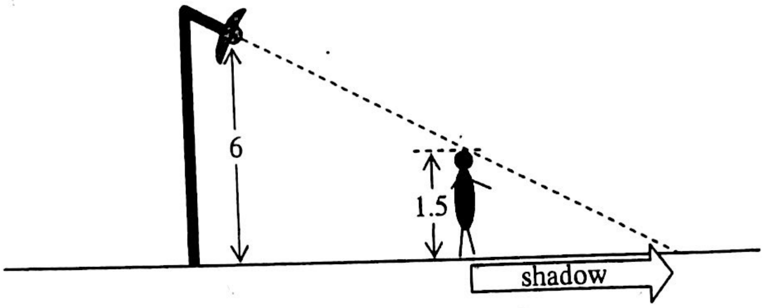
(4)



The light bulb in a lamp-post stands 6 m high.

A boy, of height 1.5 m, is walking in a straight line away from the lamp-post at constant speed of 1.5 ms^{-1}.

Determine the rate at which the length of its shadow is increasing.

(5)

Fine magnetised iron fillings are falling onto a horizontal surface forming a heap in the shape of a right circular cone of height $7x$ cm and radius x cm.

The area of the curved surface of the conical heap is increasing at the constant rate of $k \text{ cm}^2\text{s}^{-1}$, $k > 0$.

Determine the value of k, given further that when $x = 5$ the volume of the heap is increasing at the rate of $24.5 \text{ cm}^3\text{s}^{-1}$.

You may assume that the volume V and curved surface area A of a right circular cone of radius r and height h are given by

$$V = \frac{1}{3}\pi r^2 h \quad \text{and} \quad A = \pi r\sqrt{r^2 + h^2}.$$