

Created by T. Madas

INTEGRATION

BY PARTS

(student version)

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Question 1

Carry out the following integrations:

1. $\int x e^{2x} dx$

2. $\int 3x \cos 2x dx$

3. $\int x \sin 4x dx$

4. $\int -2x \sin 5x dx$

5. $\int (1-2x)e^{-x} dx$

6. $\int x^2 e^{-3x} dx$

7. $\int 16x^3 \ln x dx$

8. $\int \ln x dx$

9. $\int x \cos\left(\frac{1}{2}x\right) dx$

10. $\int (3x-1)\sin(3x-1) dx$

Question 2

Carry out the following integrations:

1. $\int 6xe^{3x} dx$

2. $\int 12x \cos 3x dx$

3. $\int x \sin 6x dx$

4. $\int -x \sin 2x dx$

5. $\int (2-x)e^{-3x} dx$

6. $\int x^2 e^{4x} dx$

7. $\int x^2 e^{-\frac{1}{2}x} dx$

8. $\int 25x^4 \ln x dx$

9. $\int 24x \cos\left(\frac{2}{3}x\right) dx$

10. $\int x^2 \sin(1-x) dx$

Question 3

Carry out the following integrations:

1. $\int \frac{1}{2} x e^{4x} dx$

2. $\int 5x \sin 4x dx$

3. $\int (2x+1) \cos 2x dx$

4. $\int -3x \cos 4x dx$

5. $\int x^2 e^{-2x} dx$

6. $\int x^2 \sin 5x dx$

7. $\int x^2 \cos \frac{1}{3} x dx$

8. $\int \frac{1}{2} x^3 \ln x dx$

9. $\int x \ln 3x dx$

10. $\int \frac{\ln x}{x^3} dx$

Question 4

Carry out the following integrations:

1. $\int x e^{5x} dx$

2. $\int 2x \cos 3x dx$

3. $\int x \sin 3x dx$

4. $\int x \sin 4x dx$

5. $\int 2 \ln x dx$

6. $\int x^2 \ln x dx$

7. $\int x \sin\left(\frac{1}{2}x\right) dx$

8. $\int x \sin(2x-1) dx$

9. $\int \frac{\ln x}{x^2} dx$

10. $\int x \sec^2 x dx$

Question 5

Carry out the following integrations:

1. $\int x^2 \sin x \, dx$

2. $\int x^3 \ln x \, dx$

3. $\int \sin x \ln(\sec x) \, dx$

4. $\int x \cos 5x \, dx$

5. $\int x^2 \sin 3x \, dx$

6. $\int 4x e^{-\frac{2}{3}x} \, dx$

7. $\int x^2 \cos\left(\frac{1}{3}x\right) \, dx$

8. $\int 2x^2 \sec^2 x \, dx$

9. $\int x^2 e^{\frac{1}{2}x} \, dx$

10. $\int x \sec x \tan x \, dx$

Question 6

Carry out the following integrations:

1. $\int x^2 e^{-\frac{1}{4}x} dx$

2. $\int x^2 e^{-x} dx$

3. $\int e^x \cos x dx$

4. $\int (\ln x)^2 dx$

5. $\int e^x \sin x dx$

6. $\int (x^3 + 5x^2 - 2)e^{2x} dx$

7. $\int x \cos^2 x dx$

8. $\int x \ln 2x^3 dx$

Question 7

Carry out the following integrations, to the answer given:

1. $\int_0^{\ln 2} x e^{2x} dx = \ln 4 - \frac{3}{4}$

2. $\int_0^{\frac{\pi}{3}} 6x \sin 3x dx = \frac{2\pi}{3}$

3. $\int_0^{\frac{\pi}{2}} x^2 \cos x dx = \frac{1}{4}(\pi^2 - 8)$

4. $\int_1^e x \ln x dx = \frac{1}{4}(e^2 + 1)$

5. $\int_0^1 4x e^{3x} dx = \frac{4}{9}(2e^3 + 1)$

6. $\int_0^{\frac{\pi}{4}} x \sin 4x dx = \frac{\pi}{16}$

7. $\int_1^2 x^3 \ln x dx = 4 \ln 2 - \frac{15}{16}$

8. $\int_0^1 x e^{-2x} dx = \frac{1}{4}(1 - 3e^{-2})$

9. $\int_0^{\frac{\pi}{4}} 12x \cos 2x dx = \frac{3}{2}(\pi - 2)$

10. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 4x \sin 2x dx = \pi - 1$

Question 8

Carry out the following integrations, to the answer given:

1. $\int_0^{\frac{\pi}{3}} x \sin 3x \, dx = \frac{\pi}{9}$

2. $\int_0^{\frac{\pi}{4}} 2x \cos 4x \, dx = -\frac{1}{4}$

3. $\int_0^{\ln 2} 4x e^{-x} \, dx = 2 - \ln 4$

4. $\int_1^e \ln x \, dx = 1$

5. $\int_0^{\frac{\pi}{2}} x \sin 2x \, dx = \frac{\pi}{4}$

6. $\int_0^{\ln 4} x e^{\frac{1}{2}x} \, dx = 8 \ln 2 - 4$

7. $\int_0^{\pi} x \cos\left(\frac{1}{4}x\right) \, dx = 2\sqrt{2}(\pi + 4) - 16$

8. $\int_0^1 (2x+1)e^{2x} \, dx = e^2$

9. $\int_{\frac{1}{e}}^1 x \ln x \, dx = \frac{1}{4}\left(\frac{3}{e^2} - 1\right)$

10. $\int_{-1}^0 3 \ln(2x+3) \, dx = \frac{3}{2}(\ln 27 - 2)$ **REQUIRES ADDITIONAL TECHNIQUES**

Question 9

Carry out the following integrations, to the answer given:

1. $\int_0^{\frac{\pi}{4}} x \sec^2 x \, dx = \frac{1}{4}(\pi - \ln 4)$

2. $\int_1^2 \frac{\ln x}{x} \, dx = \frac{1}{2}(\ln 2)^2$

3. $\int_0^{\frac{\pi}{2}} x \sin^2 x \, dx = \frac{1}{16}(\pi^2 + 4)$