Question 1 (**)

A particle P is moving on the x axis and its acceleration a ms^{-2}, t seconds after a given instant, is given by

$$a = 6t - 18, \ t \geq 0.$$

The particle is initially at the origin O, moving with a speed of 15 ms^{-1} in the positive x direction.

a) Determine the times when P is instantaneously at rest.

b) Find the distance between the points, at which P is instantaneously at rest.

Question 2 (**)

A particle P is moving on the x axis and its velocity v ms^{-1}, t seconds after a given instant, is given by

$$v = t^2 - 4t - 12, \ t \geq 0.$$

When $t = 0$, its displacement x from the origin O is 20 m.

a) Find the acceleration of P when $t = 3$.

b) Find the acceleration of P, when P is instantaneously at rest.

c) Determine the distance of P from O, when P is instantaneously at rest.

Question 3 (**+)

A particle P is moving on the x axis and its velocity v ms^{-1}, t s after a given instant, is given by

$$v = t^2(3 - t), \ t \geq 0.$$

When $t = 2$, P is observed to be 4 m from the origin O, in the positive x direction.

a) Find the acceleration of P when $t = 2$.

The particle is at instantaneous rest initially, and when $t = T$.

b) Determine the distance of P from O when $t = T$.

Question 4 (***)

A particle P is moving on the x axis and its acceleration a ms^{-2}, t seconds after a given instant, is given by

$$a = 8 - 2t, \ t \geq 0.$$

Initially, P is on the positive x axis 84 m away from the origin O, and is moving towards O with a speed of 7 ms^{-1}.

a) Find an expression for the velocity of P.

b) Calculate the maximum velocity of P.

c) Determine the times when P is instantaneously at rest.

d) Show that when $t = 12$, P is passing through O.

Question 5 (***)

A particle is moving in a straight line.

At time t s, the particle has displacement x m from a fixed origin O and is moving with velocity v ms^{-1}. When $t = 1$, $x = -5$ and $v = 1$.

The acceleration a of the particle is given by

$$a = (16 - 6t) \text{ ms}^{-2}, \ t \geq 0.$$

The particle passes through O with speed U when $t = T$, $t > 0$.

Find the possible values of U.

Question 6 (*+)**

A particle P is moving on the x axis and its displacement x m, t seconds after a given instant, is given by

$$x = 2t^3 - 3t^2 + At + B, \ t \geq 0,$$

where A and B are constants.

a) Find the value of t when the acceleration of P is zero.

When $t = 1.5$ s, P is passing through the origin O, and is moving in the negative x direction with speed 7.5 ms^{-1}.

b) Determine the value of A and the value of B.

c) Determine the time when P is instantaneously at rest.

d) Calculate as an exact surd the value of t, when P is passing through O again.

Question 7 (*+)**

A car is travelling on a straight horizontal road with constant velocity of $37.5\ \text{ms}^{-1}$.

The driver applies the brakes and the car decelerates at $(9.25-t)\ \text{ms}^{-2}$, where t s is the time since the instant when the brakes where first applied.

a) Show that while the car is decelerating its velocity is given by

$$\frac{1}{4}\left(2t^2-37t+\text{A}\right)\text{ms}^{-1}. \qquad \text{State A}$$

b) Hence find the time taken to bring the car to rest.

c) Determine the distance covered while the car was decelerating.

Question 8 (*+)**

A particle P is moving on the x axis and its velocity $v\ \text{ms}^{-1}$ in the positive x direction, t seconds after a given instant, is given by

$$v=t^2-2t-24,\ t\geq 0.$$

When $t=3$, P is observed passing through the origin.

a) Find the acceleration of P when $t=3$.

b) Determine the distance of P from O when it is instantaneously at rest.

c) Find the time at which P is passing through O again.

Question 9 (*+)**

A particle P is moving on the x axis and its velocity $v\ \text{ms}^{-1}$ in the positive x direction, t seconds after a given instant, is given by

$$v=3t^2-18t+24,\ t\geq 0.$$

a) Find the times when P is instantaneously at rest.

b) Determine the greatest speed of P in the interval $0\leq t\leq 3$.

c) Calculate the total distance covered by P in the interval $0\leq t\leq 3$.

Question 10 (****)

A particle P is moving on the x axis and its acceleration a ms^{-2}, t seconds after a given instant, is given by

$$a = 4t - 9, \; t \geq 0.$$

When $t = 1$, P is moving with a velocity of -3 ms^{-1}.

a) Find the minimum velocity of P.

b) Determine the times when P is instantaneously at rest.

c) Find the distance travelled by P in the first $4\frac{1}{2}$ seconds of its motion.

Question 11 (**+)**

A particle P is moving on the x axis and its velocity v ms^{-1}, t seconds after a given instant, is given by

$$v = \begin{cases} 6t - t^2 & 0 \leq t \leq 5 \\ 25 - 4t & t > 5 \end{cases}$$

The particle is initially at the origin O.

a) Find the greatest speed of P for $0 \leq t \leq 5$.

b) Show that the distance of P from O when $t = 5$ is 8 m. State B

c) State the time at which P is instantaneously at rest for $t > 5$.

d) Hence determine the **total distance** travelled by P during the first 10 seconds of its motion.

Question 12 (**+)**

A particle P is moving on the x axis and its velocity v ms^{-1} in the positive x direction, t seconds after a given instant, is given by

$$v = \frac{1}{2}t^2 - 3t + 4, \; t \geq 0.$$

The particle is passing through the origin when $t = 0$

Determine the displacement of the particle from the origin, when it has covered a **total distance** of 13 m.

Question 13 (**+)**

A car moving on a straight road is modelled as a particle moving on the x axis, and its acceleration a ms^{-2}, t seconds after a given instant, is given by

$$a=\begin{cases}4-\frac{1}{2}t & 0\le t\le 8\\ 0 & t>8\end{cases}$$

The car starts from rest at the origin O.

a) Find a similar expression for the velocity of the car, as that of its acceleration.

b) State the time it takes for the car to reach its maximum speed.

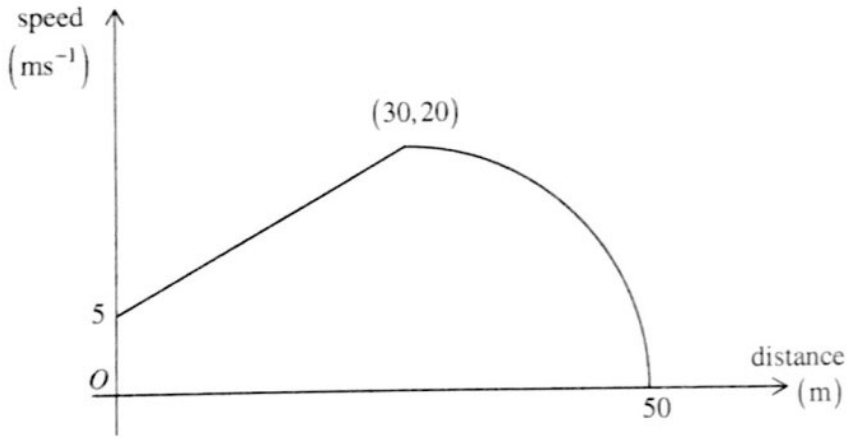
c) Show that the displacement of P from O is given by

$$x=\begin{cases}2t^2-\mathbf{C}\,t^3 & 0\le t\le 8\\ 16t-\mathbf{D} & t>8\end{cases}$$

State C, D

d) Calculate the time it takes the car to cover the first 1000 m.

Question 14 (*****)



The speed distance graph of the journey of a particle is shown above.

It consists of a straight line segment joining the point $(0,5)$ to $(30,20)$, joined to a quarter circle of radius 20. The total distance covered by the particle is 50 m.

Determine in exact form the total journey time of the particle.

You may assume without proof that

$$\int \frac{1}{\sqrt{a^2-(u+b)^2}}\,du=\arcsin\left(\frac{u-b}{a}\right)+\text{constant}$$

1) $\boxed{t = 1,\ 5}$, $\boxed{d = 32\,\text{m}}$

2) $\boxed{a = 2\ \text{ms}^{-2}}$, $\boxed{a = 8\ \text{ms}^{-2}}$, $\boxed{d = 52\,\text{m}}$

3) $\boxed{a = 0}$, $\boxed{d = 6.75\,\text{m}}$

4) $\boxed{v = -t^2 + 8t - 7}$, $\boxed{v_{\max} = 9\ \text{ms}^{-1}}$, $\boxed{t = 1,\ 7}$

5) $\boxed{U = 8,\ 24}$

6) $\boxed{t = \frac{1}{2}}$, $\boxed{A = -12}$, $\boxed{B = 18}$ $\boxed{t = 2}$, $\boxed{t = \sqrt{6}}$

7) 150 $\boxed{t = 6\ \text{s}}$, $\boxed{d = 94.5\ \text{m}}$

8) $\boxed{a = 4\,\text{ms}^{-2}}$, $\boxed{d = 36\,\text{m}}$, $\boxed{t = \sqrt{72} \approx 8.49}$

9) $\boxed{t = 2,\ 4}$, $\boxed{|v|_{\max} = 24\ \text{ms}^{-1}}$, $\boxed{d = 22\ \text{m}}$

10) $\boxed{v_{\min} = -6.125\,\text{ms}^{-1}}$, $\boxed{t = \frac{1}{2},\ 4}$, $\boxed{d = \frac{389}{24} \approx 16.21\,\text{m}}$

11) $\boxed{v_{\max} = 9\,\text{ms}^{-1}}$, $\frac{100}{3}$ $\boxed{t = \frac{25}{4} = 6.25\ \text{s}}$, $\boxed{d = \frac{775}{12} \approx 64.58\ \text{m}}$

12) $\boxed{x = \frac{35}{3}}$

13) $\boxed{v = \begin{cases} 4t - \frac{1}{4}t^2 & 0 \le t \le 8 \\ 16 & t > 8 \end{cases}}$, $\boxed{t = 8\ \text{s}}$, $\frac{1}{12}$, $\frac{128}{3}$ $\boxed{t = 65\frac{1}{6}\ \text{s}}$

14) $\boxed{t = \left(\frac{1}{2}\pi + 4\ln 2\right)\text{s}}$