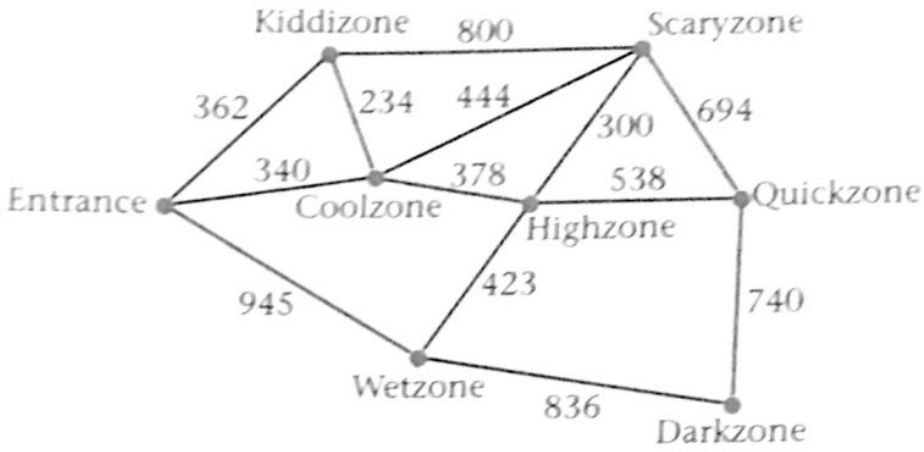
1 The network represents a theme park with seven zones. The number on each arc distance in metres.



Tramways are to be built to link the seven zones and the car park at the Entrance.

a Find a minimum connector using

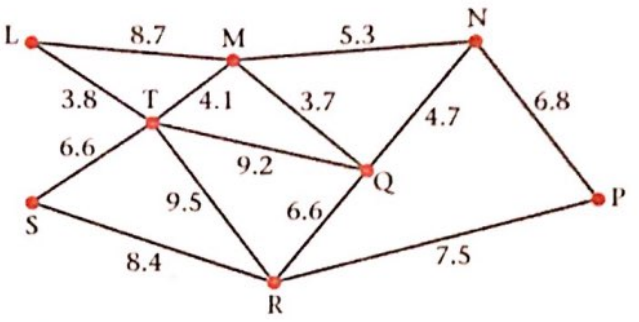
i Kruskal's algorithm,

ii Prim's algorithm, starting at the Entrance.

You must make your order of arc selection clear.

b Draw your tree and state its weight.

2 The network represents eight observation points in a wildlife reserve and the possible paths connecting them. The number on each arc is the distance, in kilometres, along that path. It is decided to link the observation points by paths, but in order to minimise the impact on the wildlife reserve, we wish to use the least total length of path.



a Find a minimum spanning tree for the network using

i Prim's algorithm, starting at L, **ii** Kruskal's algorithm.

In each case list the arcs in the order in which you consider them.

Given that paths TQ and RP already exist and so will form part of the tree,

b state which algorithm, Prim's or Kruskal's, you would select to complete the spanning tree. Give a reason for your answer.

3 *

	A	B	C	D	E	F
A	–	124	52	87	58	97
B	124	–	114	111	115	84
C	52	114	–	67	103	98
D	87	111	67	–	41	117
E	58	115	103	41	–	121
F	97	84	98	117	121	–

The table shows the distances, in mm, between six nodes A to F in a network.

a Use Prim's algorithm, starting at A, to solve the minimum connector problem for this table of distances. You must explain your method carefully and indicate clearly the order in which you selected the arcs.

b Draw a sketch showing the minimum spanning tree and find its length. E

4 *It is intended to network five computers at a large theme park. There is one computer at the office and one at each of the four different entrances. Cables need to be laid to link the computers. Cable laying is expensive, so a minimum total length of cable is required.

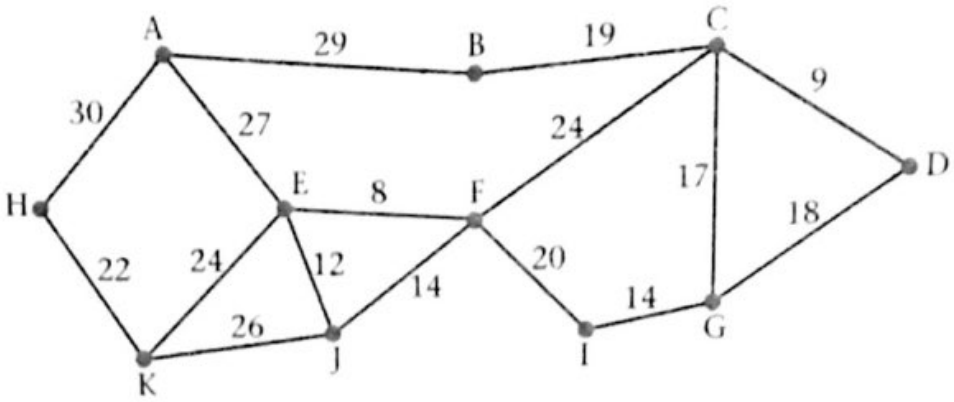
The table shows the shortest distances, in metres, between the various sites.

	Office	Entrance 1	Entrance 2	Entrance 3	Entrance 4
Office	–	1514	488	980	945
Entrance 1	1514	–	1724	2446	2125
Entrance 2	488	1724	–	884	587
Entrance 3	980	2446	884	–	523
Entrance 4	945	2125	587	523	–

a Starting at Entrance 2, demonstrate the use of Prim's algorithm and hence find a minimum spanning tree. You must make your method clear, indicating the order in which you selected the arcs in your final tree.

b Calculate the minimum total length of cable required. E

5



You are to use Kruskal's algorithm to find a minimum spanning tree for the network shown.

a i Write down the order in which you selected the arcs.

ii Sketch your minimum spanning tree.

iii State the weight of your minimum spanning tree.

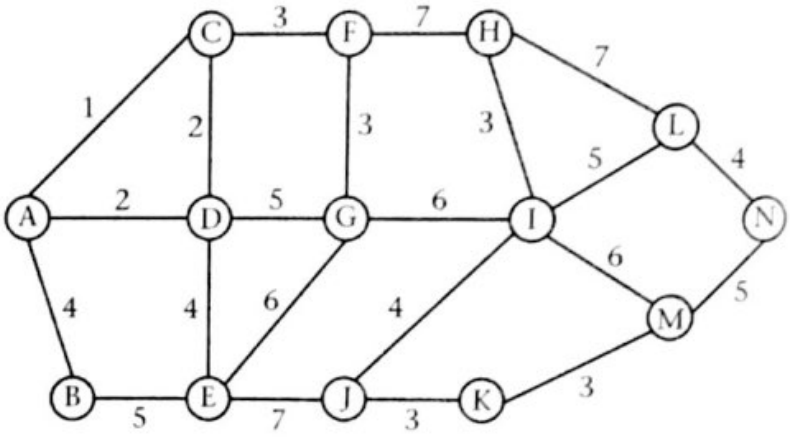
For any connected network,

E = the number of edges in the minimum spanning tree, and

V = the number of vertices in the network.

b Write down the relationship between E and V.

6 A company is to install power lines to buildings on a large industrial estate. The lines are to be laid by the side of the roads on the estate. The estate is shown as a network opposite. The buildings are designated A, B, C, ..., N and the distances between them are given in hundreds of metres. The manager wants to minimise the total length of power line to be used.



a Use Kruskal's algorithm to obtain a minimum spanning tree for the network and hence determine the minimum length of power line needed.

Owing to a change of circumstances, the company modifies its plans for the estate. The result is that the road from F to G now has a length of 700 metres.

b Determine the new minimum total length of power line.

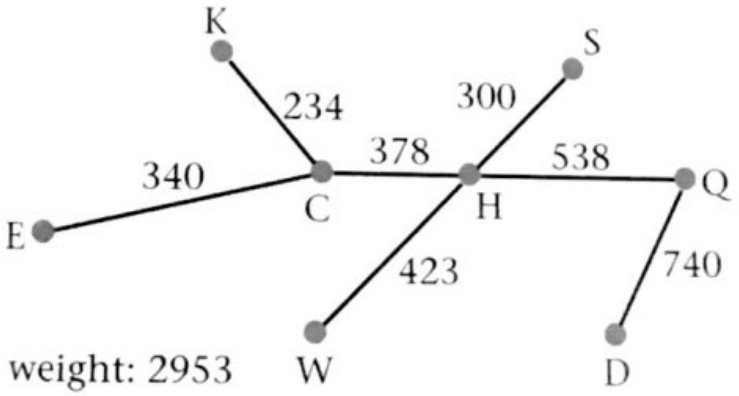
shortest route: 10

Exercise 3E

1 a i Arcs are labelled with initial letters of the nodes.
CK add to tree
SH add to tree
CE add to tree
EK reject
CH add to tree
HW add to tree
CS reject
HQ add to tree
QS reject
QD add to tree
KS reject
DW reject
EW reject

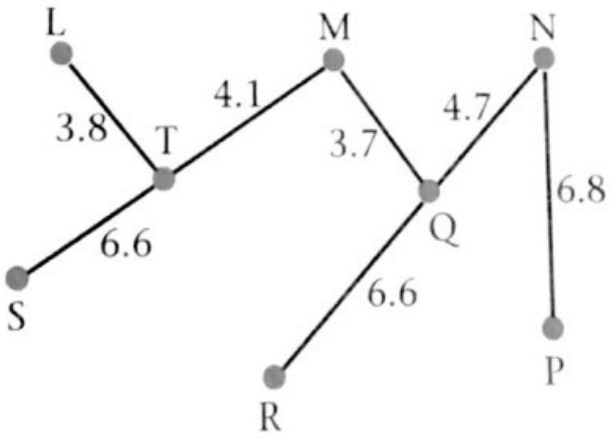
ii EC
CK
CH
HS
HW
HQ
QD

b



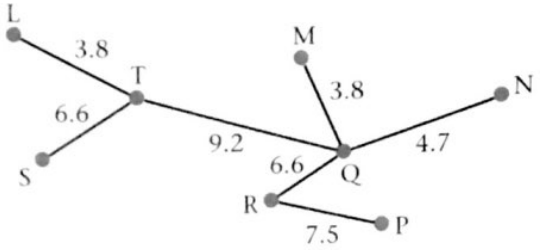
weight: 2953

2 a i LT
MT
MQ
NQ
ST
QR
NP

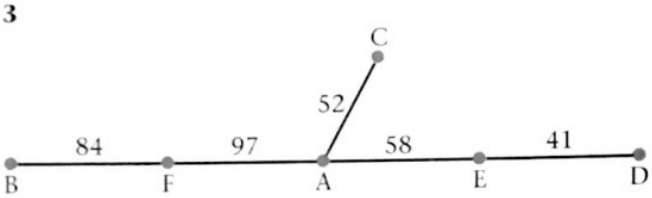


ii MQ (3.7) add to tree
LT (3.8) add to tree
MT (4.1) add to tree
NQ (4.7) add to tree
MN (5.3) reject
ST (6.6) add to tree
QR (6.6) add to tree
NP (6.8) add to tree
reject remaining arcs

b Start off the tree with QT and PR then apply Kruskal's algorithm. Prim's algorithm requires the 'growing' tree to be connected at all times. When using Kruskal's algorithm the tree can be built from non-connected sub-trees.

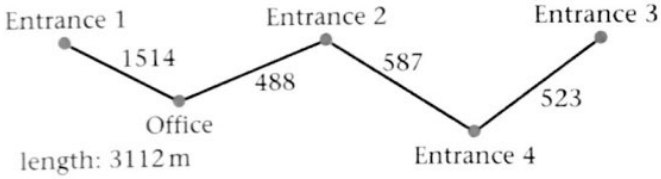


3



length 332 mm

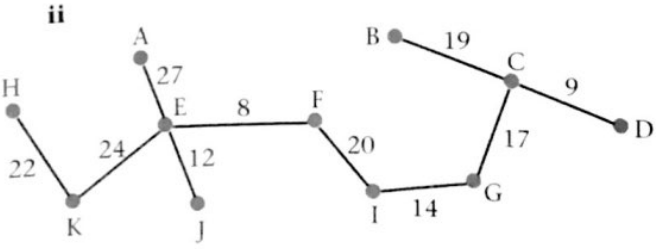
4 Arcs in order: Entrance 2–office; Entrance 2–Entrance 4; Entrance 4–Entrance 3; Office–Entrance 1



length: 3112 m

5 **i** EF (8) add to tree
CD (9) add to tree
EJ (12) add to tree
FJ (14) reject
GI (14) add to tree
CG (17) add to tree
DG (18) reject
BC (19) add to tree
FI (20) add to tree
HK (22) add to tree
EK (24) add to tree
CF (24) reject
JK (26) reject
AE (27) add to tree
AB (29) reject remaining arcs
AH (30)

ii

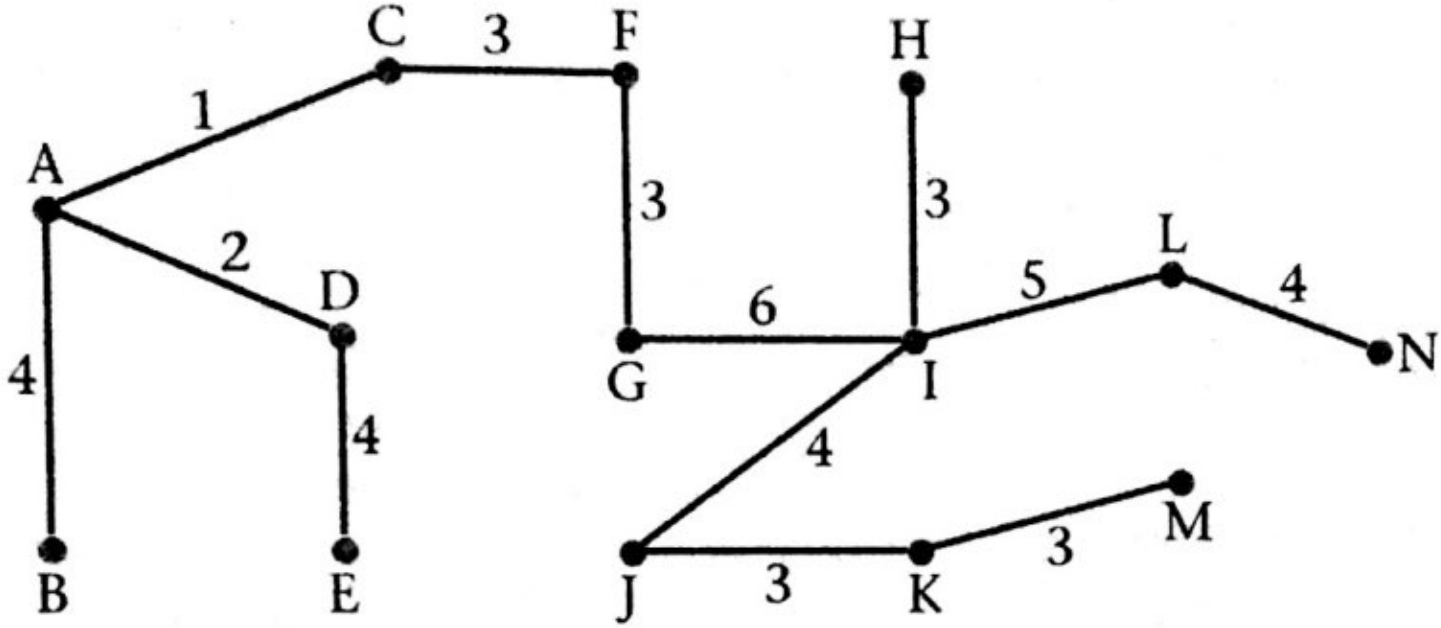


iii weight: 172

b $V = E + 1$

6 a

weight = 45 so 45 00 m needed



b weight = 47 so 4700 m