Core Mathematics C4
June 2005

1. Use the binomial theorem to expand $\sqrt{(4 - 9x)}$, $\quad |x| < \frac{4}{9}$,

in ascending powers of x, up to and including the term in x^3, simplifying each term. **(5)**

2. A curve has equation $x^2 + 2xy - 3y^2 + 16 = 0$.

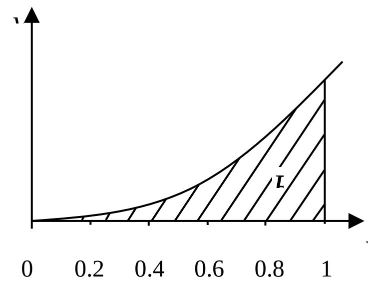
Find the coordinates of the points on the curve where $\frac{dy}{dx} = 0$. **(7)**

3. (*a*) Express $\frac{5x+3}{(2x-3)(x+2)}$ in partial fractions. **(3)**

(*b*) Hence find the exact value of $\int_2^6 \frac{5x+3}{(2x-3)(x+2)}\,dx$, giving your answer as a single logarithm. **(5)**

4. Use the substitution $x = \sin\theta$ to find the exact value of $\int_0^{\frac{1}{2}} \frac{1}{(1-x^2)^{\frac{3}{2}}}\,dx$. **(7)**

5. Figure 1 shows the graph of the curve with equation $y = xe^{2x}$, $\quad x \geq 0$.



The finite region R bounded by the lines $x = 1$, the x-axis and the curve is shown shaded in Figure 1.

(*a*) Use integration to find the exact value of the area for R. **(5)**

(*b*) Complete the table with the values of y corresponding to $x = 0.4$ and 0.8. **(1)**

x	0	0.2	0.4	0.6	0.8	1
$y = xe^{2x}$	0	0.29836		1.99207		7.38906

(*c*) Use the trapezium rule with all the values in the table to find an approximate value for this area, giving your answer to 4 significant figures. **(4)**

6. A curve has parametric equations $\quad x = 2\cot t,\ \ y = 2\sin^2 t,\ \ 0 < t \leq \frac{\pi}{2}$.

(*a*) Find an expression for $\frac{dy}{dx}$ in terms of the parameter t. **(4)**

(*b*) Find an equation of the tangent to the curve at the point where $t = \frac{\pi}{4}$. **(4)**

(*c*) Find a cartesian equation of the curve in the form $y = f(x)$. State the domain on which the curve is defined. **(4)**

7. The line l_1 has vector equation $\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$

and the line l_2 has vector equation $\mathbf{r} = \begin{pmatrix} 0 \\ 4 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$,

where λ and μ are parameters.
The lines l_1 and l_2 intersect at the point B and the acute angle between l_1 and l_2 is θ.

(*a*) Find the coordinates of B. **(4)**
(*b*) Find the value of $\cos\theta$, giving your answer as a simplified fraction. **(4)**

The point A, which lies on l_1, has position vector $\mathbf{a} = 3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$.
The point C, which lies on l_2, has position vector $\mathbf{c} = 5\mathbf{i} - \mathbf{j} - 2\mathbf{k}$.
The point D is such that $ABCD$ is a parallelogram.

(*c*) Show that $|\overrightarrow{AB}| = |\overrightarrow{BC}|$. **(3)**
(*d*) Find the position vector of the point D. **(2)**

8. Liquid is pouring into a container at a constant rate of 20 $\text{cm}^3\,\text{s}^{-1}$ and is leaking out at a rate proportional to the volume of the liquid already in the container.

(*a*) Explain why, at time t seconds, the volume, V cm^3, of liquid in the container satisfies the differential equation $\dfrac{dV}{dt} = 20 - kV$, where k is a positive constant. **(2)**

The container is initially empty.
(*b*) By solving the differential equation, show that $V = A + Be^{-kt}$,
giving the values of A and B in terms of k. **(6)**

Given also that $\dfrac{dV}{dt} = 10$ when $t = 5$,
(*c*) find the volume of liquid in the container at 10 s after the start. **(5)**

C4 Summer 2005 answers

1. $2 - \frac{9}{4}x - \frac{81}{64}x^2 - \frac{729}{512}x^3$

2. (2, -2) (-2, 2)

3. a) A = 3, B = 1 b) ln 54

4. $\frac{1}{\sqrt{3}}$

5. a) $\frac{1}{4} + \frac{1}{4}e^2$ b) 0.89022, 3.96243 c) 2.168

6. a) $\frac{-2\sin t \cos t}{\text{cosec}^2 t}$ b) $x + 2y = 4$ c) $y = \frac{8}{4 + x^2}$

7. a) (2, 2, - 2) b) $\frac{1}{3}$ d) 6**i** – 2**j** +2**k**

8. b) $V = \frac{20}{k} - \frac{20}{k}e^{-kt}$ c) $V = \frac{75}{\ln 2}$