

## TT2 Result

$$\textcircled{1} \quad a) \quad 1 + \frac{x-8}{x^2+2x-8} - \frac{2}{x+4}$$

$$\equiv \frac{(x+4)(x-2) + (x-8) - 2(x-2)}{(x+4)(x-2)}$$

$$\equiv \frac{x^2 + 2x - 8 + x - 8 - 2x + 4}{(x+4)(x-2)}$$

$$\equiv \frac{x^2 + x - 12}{(x+4)(x-2)}$$

$$\equiv \frac{(x+4)(x-3)}{(x+4)(x-2)}$$

$$\equiv \frac{x-3}{x-2}$$

$$b) \quad f'(x) = \frac{(x-2) - (x-3)}{(x-2)^2}$$

$$= \frac{1}{(x-2)^2}$$

$$(2) \quad a) \quad m = 20e^{0.02t}$$

$$t=0 \Rightarrow m=20$$

$$m=60 \Rightarrow 60 = 20e^{0.02t}$$

$$\therefore 3 = e^{0.02t}$$

$$\therefore t = \frac{1}{0.02} \ln 3 = 54.93 \text{ hours (2d.p.)}$$

$$b) \quad \frac{dm}{dt} = 20 \times 0.02 e^{0.02t} \\ = 0.4 e^{0.02t}$$

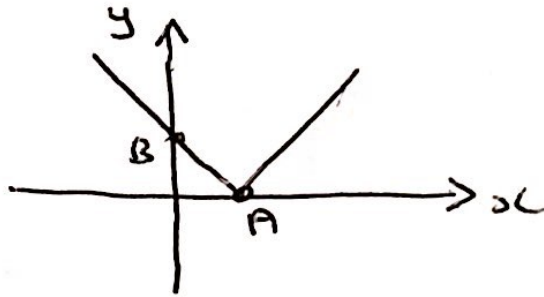
$$m=100 \Rightarrow 100 = 20e^{0.02t}$$

$$\therefore 5 = e^{0.02t}$$

$$\therefore t = \frac{1}{0.02} \ln 5$$

$$t = 80.4719 \Rightarrow \frac{dm}{dt} = 0.4 e^{0.02 \times \frac{1}{0.02} \ln 5} \\ = 0.4 \times 5 = 2 \text{ kg h}^{-1}$$

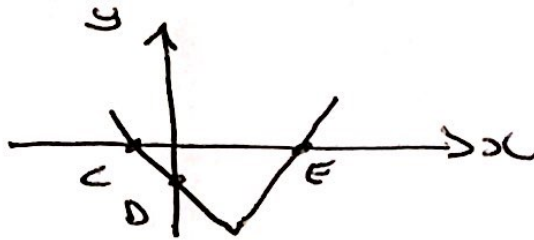
(3) a) i)



$$A(a, 0)$$

$$B(0, a)$$

ii)



$$C(a-b, 0)$$

$$D(0, a-b)$$

$$E(a+b, 0)$$

$$b) \quad |x-a| - b = \frac{1}{2}x$$

$$\therefore x-a-b = \frac{1}{2}x \quad \text{or} \quad a-x-b = \frac{1}{2}x$$

$$\therefore \frac{1}{2}x = a+b \quad \text{or} \quad a-b = \frac{3}{2}x$$

$$\therefore x = 2(a+b) \quad \text{or} \quad x = \frac{2}{3}(a-b)$$

$$\textcircled{4} \text{ a) } \int \frac{12x^2 + 50x + 2}{8x^3 + 5x^2 + 4x - 1} dx$$

$$= \frac{1}{2} \int \frac{24x^2 + 100x + 4}{8x^3 + 5x^2 + 4x - 1} dx$$

$$= \frac{1}{2} \ln |8x^3 + 5x^2 + 4x - 1| + C$$

$$\text{b) } \int \cos^2 x (1 + \sin x) dx$$

$$= \int \cos^2 x + \cos^2 x \sin x dx$$

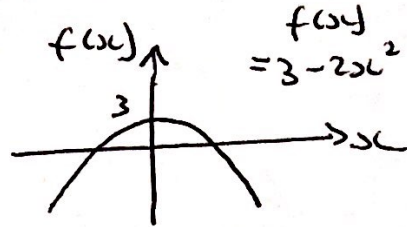
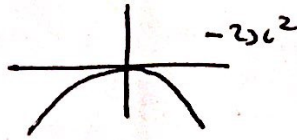
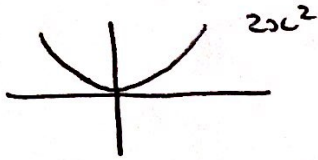
$$= \frac{1}{2} x + \frac{1}{4} \sin 2x - \frac{1}{3} \cos^3 x + C$$

$$\text{c) } \int \frac{3x}{\sqrt{x^2 - 4}} dx$$

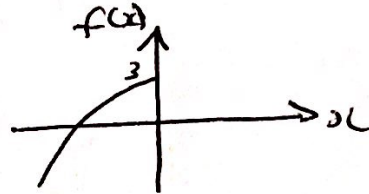
$$= 3 \int x (x^2 - 4)^{-1/2} dx$$

$$= 3 (x^2 - 4)^{1/2} + C$$

5) a)  $f(x) = 3 - 2x^2 \quad x \leq 0$



but the domain of  $f$  is  $x \leq 0$



$\therefore$  range of  $f$  is  $f(x) \leq 3$

b)  $f(f(x)) = f(3 - 2x^2)$

$$\begin{aligned}
 &= 3 - 2(3 - 2x^2)^2 \\
 &= 3 - 2(9 - 12x^2 + 4x^4) \\
 &= 3 - 18 + 24x^2 - 8x^4 \\
 &= -8x^4 + 24x^2 - 15 \quad \text{as required}
 \end{aligned}$$

$f(f(x)) = -47 \Rightarrow -47 = -8x^4 + 24x^2 - 15$

$\therefore 8x^4 - 24x^2 - 32 = 0$

$\therefore (x^2 - 4)(8x^2 + 8) = 0$

$\therefore x^2 = 4, -8$

since  $x$  is real and  $x < 0$ ,  
 $x = -2$

c)  $y = 3 - 2x^2$

$\therefore x^2 = \frac{3 - y}{2}$

$\therefore x = \pm \sqrt{\frac{3 - y}{2}}$

since  $x < 0$

$f^{-1}(y) = -\sqrt{\frac{3 - y}{2}}$

d)  $f(x) = f^{-1}(x)$

$\therefore 3 - 2x^2 = -\sqrt{\frac{3 - x}{2}}$

$\therefore 9 - 12x^2 + 4x^4 = \frac{3 - x}{2}$

$\therefore 8x^4 - 24x^2 + 18 = 3 - x$

$\therefore 8x^4 - 24x^2 + x + 15 = 0$

$\therefore x = -3/2 \dots ?$

Sorry!  
MAATS  
question  
with a  
mistake!

not  
easy to  
solve

I didn't make this so  
your Maats was out of  
46 not 50.