

① (i) Express $\frac{x}{(x+1)(x+2)}$ in partial fractions. [3]

(ii) Hence find $\int \frac{x}{(x+1)(x+2)} dx$. [2]

② (i) Express $\frac{7-2x}{(x-2)^2}$ in the form $\frac{A}{x-2} + \frac{B}{(x-2)^2}$, where A and B are constants. [3]

(ii) Hence find the exact value of $\int_4^5 \frac{7-2x}{(x-2)^2} dx$. [4]

③ (i) Use algebraic division to express $\frac{x^3 - 2x^2 - 4x + 13}{x^2 - x - 6}$ in the form $Ax + B + \frac{Cx + D}{x^2 - x - 6}$, where A, B, C and D are constants. [4]

(ii) Hence find $\int_4^6 \frac{x^3 - 2x^2 - 4x + 13}{x^2 - x - 6} dx$, giving your answer in the form $a + \ln b$. [7]

④ (i) Given that $\frac{3x+4}{(1+x)(2+x)^2} \equiv \frac{A}{1+x} + \frac{B}{2+x} + \frac{C}{(2+x)^2}$, find A, B and C . [5]

(ii) Hence or otherwise expand $\frac{3x+4}{(1+x)(2+x)^2}$ in ascending powers of x , up to and including the term in x^2 . [5]

(iii) State the set of values of x for which the expansion in part (ii) is valid. [1]

① $\frac{x}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$
 $x = A(x+2) + B(x+1)$
 $x = Ax + 2A + Bx + B$
 $x = (A+B)x + 2A+B$
 $1 = A+B$
 $0 = 2A+B$
 $(1) \Rightarrow A = 1, B = -1$
 $(2) \Rightarrow A = 1, B = -2$
 $(3) \Rightarrow A = 1, B = -1, C = 7$
 $(4) \Rightarrow A = 1, B = -2, C = 3$
 $(5) \Rightarrow A = -1, B = 2, C = 2$
 $(6) \Rightarrow A = 1, B = 2, C = 2$