

① (i) Expand $(1 + ax)^{-4}$ in ascending powers of x , up to and including the term in x^2 . [3]

(ii) The coefficients of x and x^2 in the expansion of $(1 + bx)(1 + ax)^{-4}$ are 1 and -2 respectively. Given that $a > 0$, find the values of a and b . [5]

② Find the first three terms in the expansion of $(9 - 16x)^{\frac{3}{2}}$ in ascending powers of x , and state the set of values for which this expansion is valid. [5]

③ (i) Expand $(1 + 2x)^{\frac{1}{2}}$ as a series in ascending powers of x , up to and including the term in x^3 . [3]

(ii) Hence find the expansion of $\frac{(1 + 2x)^{\frac{1}{2}}}{(1 + x)^3}$ as a series in ascending powers of x , up to and including the term in x^3 . [5]

(iii) State the set of values of x for which the expansion in part (ii) is valid. [1]

④ (i) Expand $(1 - x)^{\frac{1}{2}}$ in ascending powers of x as far as the term in x^2 . [3]

(ii) Hence expand $(1 - 2y + 4y^2)^{\frac{1}{2}}$ in ascending powers of y as far as the term in y^2 . [3]

⑤ (i) Expand $(1 + x)^{\frac{1}{3}}$ in ascending powers of x , up to and including the term in x^2 . [2]

(ii) (a) Hence, or otherwise, expand $(8 + 16x)^{\frac{1}{3}}$ in ascending powers of x , up to and including the term in x^2 . [4]

(b) State the set of values of x for which the expansion in part (ii) (a) is valid. [1]

⑤ (i) $(1+x)^{\frac{1}{3}} = 1 + \frac{1}{3}x - \frac{1}{9}x^2 + \dots$ (ii) $(8+16x)^{\frac{1}{3}} = 2(1+2x)^{\frac{1}{3}} = 2(1 + \frac{1}{3}(2x) - \frac{1}{9}(2x)^2 + \dots) = 2 + \frac{4}{3}x - \frac{8}{9}x^2 + \dots$

④ (i) $(1-x)^{\frac{1}{2}} = 1 - \frac{1}{2}x + \frac{1}{8}x^2 - \dots$ (ii) $(1-2y+4y^2)^{\frac{1}{2}} = 1 - y + \frac{2}{3}y^2 - \dots$

③ (i) $(1+2x)^{\frac{1}{2}} = 1 + x - \frac{1}{2}x^2 + \frac{1}{8}x^3 - \dots$ (ii) $\frac{(1+2x)^{\frac{1}{2}}}{(1+x)^3} = (1+x - \frac{1}{2}x^2 + \frac{1}{8}x^3 - \dots)(1-x)^3 = (1+x - \frac{1}{2}x^2 + \frac{1}{8}x^3 - \dots)(1 - 3x + 3x^2 - x^3) = 1 - 2x + \frac{5}{2}x^2 - 2x^3 + \dots$

② $(9-16x)^{\frac{3}{2}} = 27(1 - \frac{16}{9}x)^{\frac{3}{2}} = 27(1 - \frac{8}{9}x)^{\frac{3}{2}} = 27(1 - \frac{4}{3}x + \frac{2}{3}x^2 - \frac{1}{3}x^3 + \dots) = 27 - 36x + 18x^2 - 9x^3 + \dots$