(1) A curve C is given implicitly by

$$x^2 + 3xy - 2y^2 + 17 = 0.$$

a) Find the coordinates of the turning points of C.

b) Show further that

$$2 + 6\frac{dy}{dx} - 4\left(\frac{dy}{dx}\right)^2 + (3x - 4y)\frac{d^2y}{dx^2} = 0.$$

c) Hence determine the nature of these turning points.

(2) A curve C is given by the implicit equation

$$x^2 + 4xy + 2y^2 + 18 = 0.$$

a) Show clearly that

$$\frac{dy}{dx} = -\frac{x + 2y}{2x + 2y}.$$

b) Find the coordinates of the turning points of C.

c) Show further that

$$2 + 8\frac{dy}{dx} + 4\left(\frac{dy}{dx}\right)^2 + 4(x + y)\frac{d^2y}{dx^2} = 0.$$

d) Hence determine the nature of these turning points.

(3) A curve C is given implicitly by $2x^2 + xy - y^2 - 4x - y + 20 = 0$.

a) Show clearly that $\frac{dy}{dx} = \frac{4x + y - 4}{2y - x + 1}$.

b) Find the coordinates of the turning points of C.

c) Show further that $4 + 2\frac{dy}{dx} - 2\left(\frac{dy}{dx}\right)^2 + (x - 2y - 1)\frac{d^2y}{dx^2} = 0$.

d) Hence determine the nature of these turning points.

④ A curve C has implicit equation

$$x^2 - 2y^2 + 4xy - 4x - 6y + 4 = 0.$$

a) Show clearly that

$$\frac{dy}{dx} = \frac{2y + x + a}{2y - 2x + b},$$

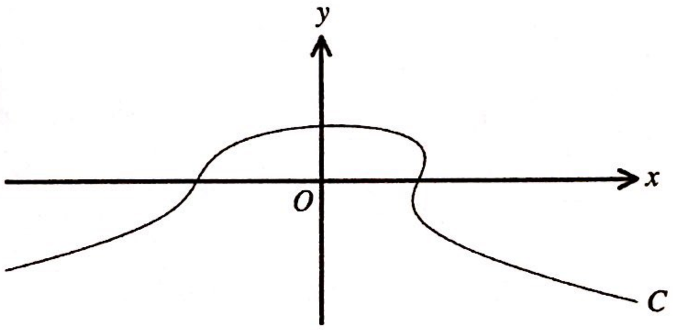
where a and b are integers to be found.

The straight line l_1 with equation $y = 2x - 3$ is a tangent to C at the point P.

The straight line l_2 is parallel to l_1 and is also a tangent to C at a different point Q.

b) Find an equation of l_2.

⑤



The figure above shows part of the curve C with equation

$$x^2 + 2x + y^3 = 63 + xy.$$

a) Show clearly that

$$\frac{dy}{dx} = \frac{y - 2x - 2}{3y^2 - x}$$

b) Show further that C has only one stationary point at $(1,4)$.

① $\boxed{(3,-2)\ \&\ (-3,2)}$, $\boxed{\text{max at } (3,-2)\ \&\ \text{min at } (-3,2)}$

② $\boxed{(-6,3)\ \&\ (6,-3)}$, $\boxed{\text{max at } (6,-3)\ \&\ \text{min at } (-6,3)}$

③ $\boxed{(2,-4)\ \&\ (0,4)}$, $\boxed{\text{max at } (2,-4)\ \&\ \text{min at } (0,4)}$

④ $\boxed{a=-2,\ b=3}$, $\boxed{y=2x-\frac{10}{3}}$