TRACKING TEST 2 RESIT
Time Allowed: 1 hour

1) a) Show clearly that $1 + \frac{x-8}{x^2+2x-8} - \frac{2}{x+4} \equiv \frac{x-p}{x-q}$, where p and q are integers **(5)**

b) Hence find an expression for f'(x), simplifying your answer as far as possible. **(4)**

2) During a chemical process, the mass of a substance, m kg, at time t hours grows exponentially according to the formula

$$m = 20e^{0.02t}, \quad t \geq 0$$

a) Find the time taken for the substance to increase to three times its initial mass. **(5)**

b) Calculate the rate of change of m when m = 100 **(4)**

3) Given that a and b are constants and that 0 < a < b,

a) On separate diagrams, sketch the graph with equation

i) y = |x - a| **(2)**

ii) y = |x - a| - b **(4)**

Show on each sketch the co-ordinates of each point at which the graph crosses or meets the axes.

b) Solve, for x, the equation |x – a| - b = ½ x **(4)**

4. Carry out the following integrations

a) $\int \frac{12x^2+5x+2}{8x^3+5x^2+4x-1} dx$ **(3)**

b) $\int \cos^2 x \, (1 + \sin x) \, dx$ **(3)**

c) $\int \frac{3x}{\sqrt{x^2-4}} \, dx$ **(3)**

5. The function f(x) is defined by $f(x) = 3 - 2x^2, \quad x \epsilon R, x \leq 0$

a) State the range of f **(2)**

b) Show that $ff(x) = -8x^4 + 24x^2 - 15$ and hence solve the equation $ff(x) = -47$ **(4)**

c) Find an expression for the inverse function $f^{-1}(x)$ **(3)**

d) Solve the equation $f(x) = f^{-1}(x)$ **(4)**

Answers

1) a) $\frac{x-3}{x-2}$ b) $\frac{1}{(x-2)^2}$
2) a) 54.93 hours b) 2 $kg\ h^{-1}$
3) a) i) (a,0) and (0,a)
   ii) (0,a-b), (a+b,0) (a-b,0) Graph must cross y axis at a negative point.
   b) x=2(a+b), $\frac{2}{3}$(a – b)
4) a) ½ ln$|x^3 + 5x^2 + 4x - 1| + c$
   b) $\frac{1}{2}x + \frac{1}{4}\sin 2x - \frac{1}{3}\cos^3 x + c$
   c) $3\sqrt{x^2 - 4} + c$
5) a) $f(x) \leq 3$
   b) x=-2
   c) $-\sqrt{\frac{3-x}{2}}$
   d) $-\frac{3}{2}$