

Numerical Methods

1. $g(x) = e^{x-1} + x - 6$
- (a) Show that the equation $g(x) = 0$ can be written as
 $x = \ln(6 - x) + 1, \quad x < 6.$ (2)
- The root of $g(x) = 0$ is α .
The iterative formula $x_{n+1} = \ln(6 - x_n) + 1, \quad x_0 = 2.$
is used to find an approximate value for α .
- (b) Calculate the values of x_1, x_2 and x_3 to 4 decimal places. (3)
- (c) By choosing a suitable interval, show that $\alpha = 2.307$ correct to 3 decimal places. (3)
2. The function f is defined by $f(x) = \ln(4 - 2x), \quad x < 2$ and $x \in \mathbb{R}.$
- (a) Show that the inverse function of f is defined by
 $f^{-1}(x) = 2 - \frac{1}{2}e^x$
and write down the domain of $f^{-1}.$ (4)
- (b) Write down the range of $f^{-1}.$ (1)
- (c) Sketch the graph of $y = f^{-1}(x).$ State the coordinates of the points of intersection with the x and y axes. (4)
- The graph of $y = x + 2$ crosses the graph of $y = f^{-1}(x)$ at $x = k.$
The iterative formula $x_{n+1} = -\frac{1}{2}e^{x_n}, \quad x_0 = -0.3,$
is used to find an approximate value for $k.$
- (d) Calculate the values of x_1 and $x_2,$ giving your answer to 4 decimal places. (2)
- (e) Find the values of k to 3 decimal places. (2)

2.

(a) $0 = e^{x-1} + x - 6 \Rightarrow x = \ln(6 - x) + 1$

M1A1*

(2)

(b) Sub $x_0 = 2$ into $x_{n+1} = \ln(6 - x_n) + 1 \Rightarrow x_1 = 2.3863$
AWRT 4 dp. $x_2 = 2.2847$ $x_3 = 2.3125$

M1, A1

A1

(3)

(c) Chooses interval $[2.3065, 2.3075]$

M1

$g(2.3065) = -0.0002(7)$, $g(2.3075) = 0.004(4)$

dM1

Sign change, hence root (correct to 3dp)

A1

(3)

6.

(a) $y = \ln(4 - 2x)$

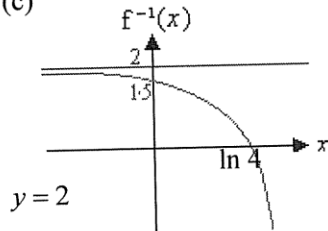
$e^y = 4 - 2x$ leading to $x = 2 - \frac{1}{2}e^y$ Changing subject and removing ln

$y = 2 - \frac{1}{2}e^x \Rightarrow f^{-1} \mapsto 2 - \frac{1}{2}e^x$ *

Domain of f^{-1} is \square

(b) Range of f^{-1} is $f^{-1}(x) < 2$ (and $f^{-1}(x) \in \square$)

(c)



Shape

B1

1.5

B1

ln 4

B1

(4)

(d) $x_1 \approx -0.3704, x_2 \approx -0.3452$

If more than 4 dp given in this part a maximum on one mark is lost.
Penalise on the first occasion.

cao

B1, B1

(2)

(e) $x_3 = -0.354\ 030\ 19 \dots$

$x_4 = -0.350\ 926\ 88 \dots$

$x_5 = -0.352\ 017\ 61 \dots$

$x_6 = -0.351\ 633\ 86 \dots$

$k \approx -0.352$

Calculating to at least x_6 to at least four dp

M1

cao

A1

(2)
[13]