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Question 1 (**)

$$x^3+10x-4=0.$$

a) Show that the above equation has a root α, which lies between 0 and 1.

The recurrence relation

$$x_{n+1}=\frac{4-x_n^3}{10}$$

starting with $x_0=0.3$ is to be used to find α.

b) Find, to 4 decimal places, the value of x_1, x_2, x_3 and x_4.

c) By considering the sign of an appropriate function $f(x)$ in a suitable interval, show clearly that $\alpha=0.39389$, correct to 5 decimal places.

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Question 2 (**)

$$e^{-x}+\sqrt{x}=2$$

a) Show that the above equation has a root α, which lies between 3 and 4.

The recurrence relation

$$x_{n+1}=\left(2-e^{-x_n}\right)^2$$

starting with $x_0=4$ is to be used to find α.

b) Find, to 3 decimal places, the value of x_1, x_2 and x_3.

c) By considering the sign of an appropriate function $f(x)$ in a suitable interval, show clearly that $\alpha=3.9211$, correct to 4 decimal places.

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Question 3 (**)

$$e^{3x} = x + 20$$

a) Show that the above equation has a root α between 1 and 2.

The recurrence relation

$$x_{n+1} = \frac{1}{3}\ln(x_n + 20)$$

starting with $x_0 = 1.5$ is to be used to find α.

b) Find to 4 decimal places, the value of x_1, x_2 and x_3.

c) By considering the sign of an appropriate function $f(x)$ in a suitable interval, show clearly that $\alpha = 1.0151$, correct to 4 decimal places.

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Question 4 (+)**

$$f(x)=4x-3\sin x-1, \quad 0\le x\le 2\pi .$$

a) Show that the equation $f(x)=0$ has a solution α in the interval $(0.7,0.8)$.

An iterative formula, of the form given below, is used to find α.

$$x_{n+1}=A+B\sin x_n, \quad x_1=0.75,$$

where A and B are constants.

b) Find, to 5 decimal places, the value of x_2, x_3, x_4 and x_5.

c) By considering the sign of $f(x)$ in a suitable interval show clearly that $\alpha=0.775$, correct to 3 decimal places.