Question 2 (**)

A new antibiotic is tested by spraying it on a lab dish covered in bacteria.

Initially 12000 bacteria were placed on the dish and 24 hours later this number has fallen to 2000.

The number of bacteria N on this lab dish reduces according to the equation

$$N = Ae^{-kt}, \ t \geq 0,$$

where t is the time in hours since the bacteria were first placed on the dish and, A and k are positive constants.

a) Show that $k = 0.07466$, correct to 4 significant figures.

b) Find the value of t when the bacteria will reach 1000.

$\boxed{}$, $\boxed{t = 33.3}$

Question 4 (**)

A preservation programme, for elephants in Africa, was introduced 8 years ago. The elephants were then released to the wild. Let t be the number of years since the start of the programme.

The population of elephants P, is given by

$$P = 400e^{\frac{1}{12}(t-8)}, \ t \geq 0.$$

Assuming that P can be treated as a continuous variable, find …

a) … the number of elephants when the programme started.

b) … the number of elephants released to the wild.

c) … the value of t when the number of elephants will reach 1000.

$\boxed{}$, $\boxed{205}$, $\boxed{400}$, $\boxed{t = 19}$

Question 5 (**)

The value £ V of a certain model of car, t years after it was purchased, is given by

$$V = Be^{-kt}, \ t > 0$$

where B and k are positive constants.

The value of the car when new was £21000 and after five years it dropped to £5000.

Find the value of B and the value of k.

$\boxed{B = 21000}$, $\boxed{k = 0.2870}$

Question 15 (***)

A liquid is cooling down and its temperature θ °C satisfies

$$\theta = 20 + 30e^{-\frac{t}{20}}, \ t > 0$$

where t is the time in minutes after a given instant.

Find the value of t when the temperature of the liquid has reduced to half its initial temperature.

$\boxed{t = 35.8}$

Question 16 (***)

A car tyre develops a puncture.

The tyre pressure P, measured in suitable units known as p.s.i., t minutes after the tyre got punctured is given by the expression

$$P = 8 + 32e^{-kt}, \ t > 0,$$

where k is a positive constant.

a) State the tyre pressure when the tyre got punctured.

The tyre pressure halves 2 minutes after the puncture occurred.

b) Show that $k = 0.4904$, correct to 4 significant figures.

c) Calculate the time it takes for the tyre pressure to drop to 12 p.s.i.

d) Find the rate at which the pressure of the tyre is changing one minute after the puncture occurred.

$\boxed{}$, $\boxed{P = 40}$, $\boxed{t = 4.24}$, $\boxed{-9.61 \text{ p.s.i/min}}$

Question 17 (***)

The population P, in thousands, of a colony of rabbits in time t years after a certain instant, is given by

$$P = 5 + ae^{-bt}, \ t \geq 0$$

where a and b are positive constants.

It is given that the initial population is 8 thousands rabbits, and one year later this population has reduced by 2 thousands.

a) Find the value of a and the value of b.

b) Explain mathematically, why the population can never reach 1000, according to this model.

$\boxed{a = 3}$, $\boxed{b = \ln 3}$

Question 48 (****)

An area is to be replanted with eucalyptus trees after a large fire.

The height, H m, of one such tree is given by the formula

$$H = 25 - 24e^{-0.1t}, \ t > 0,$$

where t is the time in years since the tree was planted.

a) State the height of a newly planted tree.

b) Find the height of a tree, after 2 years.

c) Calculate, to the nearest integer, the value of t when the height of a tree has reached 80% of its eventual height.

$\boxed{H_0 = 1 \text{ m}}$, $\boxed{H \approx 5.35 \text{ m}}$, $\boxed{t \approx 16}$

Question 49 (****)

A radioactive substance decays so that its mass, m kg, at time t years from now, satisfies the exponential equation

$$m = 400e^{-0.05t}, \ t \geq 0.$$

a) Find the time it takes for the substance to halve its mass.

b) Determine the **exact** value of t when the radioactive substance is decaying at the rate of 5 kg per year, giving the answer in terms of $\ln 2$.

$\boxed{t = 20\ln 2 \approx 13.86 \text{ years}}$, $\boxed{t = 40\ln 2}$

Question 53 (****)

Water is heated in a kettle which is kept in a kitchen. The kitchen is kept at a constant temperature, T_0.

The temperature, T °C, of the water in the kettle satisfies

$$T = 95 - 75e^{-t}, \quad t \geq 0,$$

where t is the time in minutes since the kettle was switched on.

a) Find the time it takes for the water in the kettle to reach a temperature of 85 °C.

b) Determine the initial rate of the temperature rise of the water in the kettle.

Once the water has reached a temperature of 85 °C the kettle is switched off and is allowed to cool. Its temperature is now given by

$$T = 15 + Ae^{-kt}, \quad t \geq 0,$$

where A and k are positive constants, and t now represents the time in minutes since the kettle was switched off.

c) Find the value of A.

d) State, with a reason, the constant temperature of the kitchen, T_0.

$\boxed{}$, $\boxed{t = 2.01 \text{ minutes}}$, $\boxed{75\,°\text{C/min}}$, $\boxed{A = 70}$, $\boxed{T_0 = 15}$

Question 100 (****+)

The amount X milligrams, of an anaesthetic drug in the bloodstream of a patient, is given by

$$X = De^{-0.2t}, \ t > 0$$

where D is the dose, in milligrams, of the anaesthetic administered and t is the time in hours since the dose was administered.

A patient undergoing an operation is given an initial dose of 20 milligrams.

This patient will remain asleep if there are more than 12 milligrams of anaesthetic in his bloodstream.

a) Show that one hour later $X = 16.37$, correct to two decimal places.

b) Show, by calculation, that two hours after the initial dose was administered, the patient should still be asleep.

Two hours after the initial dose was administered a further dose of **10** milligrams is given to the patient.

c) Find the amount of the anaesthetic in the patient's bloodstream one hour after the second dose is given.

No more anaesthetic is given and the operation lasts for 4 hours.

d) Show by solving a relevant equation that the patient should "wake up" approximately 80 minutes after the end of his operation.

$\boxed{}$, $\boxed{X = 19.16}$

Question 103 (*****)

On the 1st January 2000 a rare stamp was purchased at an auction for £16000 and by the 1st January 2010 its value was four times as large as its purchase price.

The future value of this stamp, £V, t years after the 1st January 2000 is modelled by the equation

$$V = A\mathrm{e}^{pt}, \; t \geq 0,$$

where A and p are positive constants.

On the 1st January 1990 a different stamp was purchased for £2.

The future value of this stamp, £U, t years after the 1st January 1990 is modelled by the equation

$$U = B\mathrm{e}^{2pt}, \; t \geq 0,$$

where B is a positive constant.

Determine the year, during which the two stamps will achieve the same value, according to their modelling equations.

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