DIFFERENTIATION

(1) Differentiate each of the following expressions with respect to x, simplifying the final answer as far as possible.

a) $y = \sec^2 x$.

b) $y = x(1-2x)^6$.

c) $y = \dfrac{\sin x}{2-\cos x}$.

(2) A curve has equation

$$y = xe^{2x}, \quad x \in \mathbb{R}.$$

Show clearly that

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0.$$

(3) The point A, where $x = \frac{1}{2}$, lies on the curve with equation

$$y = e^{2x} + \frac{2}{x}, \quad x \neq 0.$$

Show that an equation of the tangent to the curve at A is given by

$$y = (2e-8)x + 8.$$

(4) Given that

$$y = 2\sin x \tan x$$

find the exact value of $\dfrac{dy}{dx}$ at $x = \dfrac{\pi}{3}$.

(5) Differentiate each of the following expressions with respect to x, simplifying the final answers where possible.

a) $y=\sqrt{x^2-1}$

b) $y=x^4\ln x$

c) $y=\dfrac{e^x-1}{e^x+1}$

(6) The point A, where $x=2$, lies on the curve with equation

$$f(x)=x\ln x,\ x>0.$$

Find an equation of the tangent to the curve at A, giving the answer in the form $y=mx+c$, where m and c are exact constants.

(7) A curve C has equation

$$x=y^2\ln y,\ y>0.$$

Show that an equation of the normal to C at the point where $y=e$ is

$$y+3ex=e\left(3e^2+1\right).$$

(8) $$f(x)=5\ln x+\frac{1}{x},\ x>0.$$

a) Solve the equation

$$f'(x)=0.$$

b) Hence write down the y coordinate of the turning point of $f(x)$ in the form $k-k\ln k$, where k is an integer.

c) Find $f''(x)$ and use it to determine the nature of the turning point of $f(x)$.

(9) The curve C has equation

$$y = x\sqrt{\ln x}, \quad x > 1.$$

a) Find an expression for $\frac{dy}{dx}$.

b) Show that an equation of the tangent to the curve at the point where $x = e^4$ is

$$4y = 9x - e^4.$$

(10)

$$y = 2\left\{e^{2x} + 3\ln\left[x + \left(e^x + 1\right)^2\right]\right\}^2.$$

Show that the value of $\frac{dy}{dx}$ at $x = 0$ is $23(1 + 6\ln 2)$

(1) $\frac{dy}{dx} = 2\sec^2 x \tan x$, $\frac{dy}{dx} = (14x-1)(2x-1)^5 = (1-14x)(1-2x)^5$, $\frac{dy}{dx} = \frac{2\cos x - 1}{(2-\cos x)^2}$

(4) $\frac{dy}{dx} = 5\sqrt{3}$

(5) $\frac{dy}{dx} = x\left(x^2-1\right)^{-\frac{1}{2}}$, $\frac{dy}{dx} = 4x^3 \ln x + x^3$, $\frac{dy}{dx} = \frac{2e^x}{\left(e^x+1\right)^2}$

(6) $y = x(1+\ln 2) - 2$

(8) $x = \frac{1}{5}$, $y = 5 - 5\ln 5$ $f''(x) = \frac{2}{x^3} - \frac{5}{x^2}$, $f''\left(\frac{1}{5}\right) = 125 > 0$ so minimum

(9) $\frac{dy}{dx} = \sqrt{\ln x} + \frac{1}{2\sqrt{\ln x}}$