1. (a) Using the identity for $\cos(A+B)$, prove that $\cos\theta \equiv 1 - a\sin^2(\tfrac{1}{2}\theta)$.

State a

(b) Prove that $1 + \sin\theta - \cos\theta \equiv p\sin(\tfrac{1}{2}\theta)[\cos(\tfrac{1}{2}\theta) + \sin(\tfrac{1}{2}\theta)]$.

State p

(c) Hence, or otherwise, solve the equation

$$1 + \sin\theta - \cos\theta = 0, \quad 0 \le \theta < 2\pi.$$

2. $$f(x) = x + \frac{3}{x-1} - \frac{12}{x^2+2x-3}, x \in \mathbb{R}, x > 1.$$

(a) Show that $f(x) = \dfrac{x^2+3x+q}{x+3}$

State q

(b) Solve the equation $f'(x) = \frac{22}{25}$.

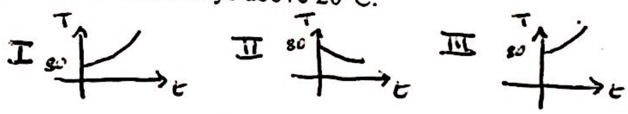
3. As a substance cools its temperature, T °C, is related to the time (t minutes) for which it has been cooling. The relationship is given by the equation

$$T = 20 + 60e^{-0.1t}, \quad t \ge 0.$$

(a) Find the value of T when the substance started to cool.

(b) Explain why the temperature of the substance is always above 20°C.

(c) Sketch the graph of T against t.



(d) Find the value, to 2 significant figures, of t at the instant $T = 60$.

(e) Find $\dfrac{dT}{dt}$.

(f) Hence find the value of T at which the temperature is decreasing at a rate of 1.8 °C per minute.

4. (i) Given that $y = \tan x + 2\cos x$, find the exact value of $\dfrac{dy}{dx}$ at $x = \dfrac{\pi}{4}$.

(ii) Given that $x = \tan\tfrac{1}{2}y$, prove that $\dfrac{dy}{dx} = \dfrac{r}{1+x^2}$.

State r

(iii) Given that $y = e^{-x}\sin 2x$, show that $\dfrac{dy}{dx}$ can be expressed in the form $R\,e^{-x}\cos(2x+\alpha)$. Find, to 3 significant figures, the values of R and α, where $0 < \alpha < \dfrac{\pi}{2}$.

Solve these equations for $0° \leq x \leq 360°$

1) $\cos 2x + \cos x + 1 = 0$

2) $\cos x = \sin\left(\frac{x}{2}\right)$

3) $\sin 2x \cos x + \sin^2 x = 1$

4) $2 \sin x\,(5 \cos 2x + 1) = 3 \sin 2x$

5) $3 \cot 2x + \cot x = 1$