1 Given that

$$y=\left(2+e^{3x}\right)^{\frac{3}{2}}$$

find the value of $\frac{dy}{dx}$ at $x=\frac{1}{3}\ln 2$.

2 A cup of coffee is cooling down in a room.

The temperature T °C of the coffee, t minutes after it was made is modelled by

$$T=20+50e^{-\frac{t}{15}},\ t>0.$$

a) State the temperature of the coffee when it was first made.

b) Find the temperature of the coffee, after 30 minutes.

c) Calculate, to the nearest minute, the value of t when the temperature of the coffee has reached 35°C.

3 Find, in exact form where appropriate, the solution of each of the following equations.

a) $4-3e^{2x}=3$

b) $\ln(2w+1)=1+\ln(w-1)$

4 The point A, where $x=1$, lies on the curve with equation

$$f(x)=(x+1)\ln x,\ x>0.$$

Find an equation of the normal to the curve at A.

5 Differentiate each of the following expressions with respect to x, simplifying the final answers as far as possible

N.B.

a) $y=\frac{4}{(2x-1)^2}$.

b) $y=x^3e^{-2x}$.

c) $y=\frac{2x^2+1}{3x^2+1}$.

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6

The point P, where $x=2$, lies on the curve with equation

$$f(x)=\ln\left(x^2+4\right).$$

Show that an equation of the normal to the curve at P, is given by

$$y+2x=a+3\ln b.$$ State a and b

7

The curve C has equation

$$y=\frac{x}{1+\ln x}, \ x>0, \ x\neq e^{-1}.$$

Show that C has a single stationary point and find its coordinates.

8

The value £ V of a certain model of car, t years after it was purchased, is given by

$$V=Be^{-kt}, \ t>0$$

where B and k are positive constants.

The value of the car when new was £21000 and after five years it dropped to £5000.

Find the value of B and the value of k.

9

$$f(x)=2-\frac{x^2}{3}+\ln\left(\frac{x}{4}\right), \ x>0.$$

a) Find an expression for $f'(x)$.

b) Find β in exact surd form, such that $f'(\beta)=0$.

10

The curve C has equation

$$y=xe^{-\frac{1}{2}x^2}, \ x\in\mathbb{R}.$$

a) Find an expression for $\frac{dy}{dx}$.

b) Find the exact coordinates of the turning points of C.

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11 A preservation programme, for elephants in Africa, was introduced 8 years ago. The elephants were then released to the wild. Let t be the number of years since the start of the programme.

The population of elephants P, is given by

$$P = 400e^{\frac{1}{12}(t-8)}, \; t \geq 0.$$

Assuming that P can be treated as a continuous variable, find ...

a) ... the number of elephants when the programme started.

b) ... the number of elephants released to the wild.

c) ... the value of t when the number of elephants will reach 1000.

12 Find, in exact form where appropriate, the solution of each of the following equations.

a) $e^{2x} = 9$

b) $\ln(4-y) = 2$

c) $\ln t + \ln 3 = \ln 12$

13 The point P, where $x = \pi$, lies on the curve with equation

$$f(x) = e^x \sin 2x, \; 0 \leq x < 2\pi.$$

Show that an equation of the normal to the curve at P, is given by

$$x + 2ye^{\pi} = a$$ State a

14 A curve C has equation

$$y = \sqrt{x^2+1}, \; x \in \mathbb{R}.$$

Show that an equation of the normal to C at the point where $x = 1$ is given by

$y = a(2-x)$ State a

15 The curve C has equation

$$y = x\ln x, \; x > 0.$$

Find the exact coordinates of the turning point of C.

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(16) It is given that

$$\frac{7}{4}\ln 16-\frac{2}{3}\ln 8 \equiv k\ln 2 .$$

Determine the value of k.

(17) A microbiologist models the population of bacteria in culture by the equation

$$P=1000-950e^{-\frac{1}{2}t},\ t>0$$

where P is the number of bacteria in time t hours.

a) Find the initial number of bacteria in the culture.

b) Show mathematically that the limiting value for P is a   State a

c) Find the value of t when $P=500$.

(18) A curve has equation

$$y=Ae^{kx},$$

where A and k are non zero constants.

The curve passes through the points $(0,4)$ and $(12,16)$.

a) Find the value of A and the exact value of k.

b) Determine the value of y when $x=30$

(19) Solve the equation

$$5+e^{2x-4}=7$$

giving the answer in the form $k+\ln\sqrt{k}$, where k is an integer.

(20) A curve C has equation

$$y=xe^{2x},\ x\in\mathbb{R}.$$

Show that an equation of the tangent to C at the point where $x=\frac{1}{2}$ is

$ay=e(bx+c)$

State a, b and c

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