

Simplify each of these expressions

1) $1 + \tan^2 \frac{1}{2}\theta$

2) $(\sec\theta-1)(\sec\theta+1)$

3) $\tan^2\theta(\operatorname{cosec}^2\theta-1)$

4) $(\sec^2\theta - 1)\cot\theta$

5) $(\operatorname{cosec}^2\theta - \cot^2\theta)^2$

6) $2 - \tan^2\theta + \sec^2\theta$

1) $\cot\theta = -\sqrt{3}$ ($90^\circ < \theta < 180^\circ$) Find the exact value of $\sin\theta$

2) $\cot\theta = -\sqrt{3}$ ($90^\circ < \theta < 180^\circ$) Find the exact value of $\cos\theta$

3) $\tan\theta = \frac{3}{4}$ ($180^\circ < \theta < 270^\circ$) Find the exact value of $\sec\theta$

4) $\tan\theta = \frac{3}{4}$ ($180^\circ < \theta < 270^\circ$) Find the exact value of $\cos\theta$

5) $\tan\theta = \frac{3}{4}$ ($180^\circ < \theta < 270^\circ$) Find the exact value of $\sin\theta$

Prove the following identities. At least one of them is not true. In these cases, state the correct Right Hand Side

1) $\sec^4\theta - \tan^4\theta \equiv \sec^2\theta - \tan^2\theta$

2) $\operatorname{cosec}^2\theta - \sin^2\theta \equiv \cot^2\theta + \sin^2\theta$

3) $\sec^2\theta(\cot^2\theta - \cos^2\theta) \equiv \cot\theta$

4) $1 - \cos^2\theta \equiv (\sec^2\theta - 1)(1 - \sin^2\theta)$

Solve the following equations. Give exact answers if possible. If not possible, give answers to 3 s.f.

1) $\sec^2 \theta = 3 \tan \theta$, $0 \leq \theta \leq 360^\circ$

2) $\tan^2 \theta - 2 \sec \theta + 1 = 0$, $-\pi \leq \theta \leq \pi$

3) $\operatorname{cosec}^2 \theta + 1 = 3 \cot \theta$, $-180 \leq \theta \leq 180^\circ$

4) $\cot \theta = 1 - \operatorname{cosec}^2 \theta$, $0 \leq \theta \leq 2\pi$

5) $3 \sec \frac{1}{2} \theta = 2 \tan^2 \frac{1}{2} \theta$, $0 \leq \theta \leq 360^\circ$

6) $(\sec \theta - \cos \theta)^2 = \tan \theta - \sin^2 \theta$, $0 \leq \theta \leq \pi$

7) $\tan^2 2\theta = \sec 2\theta - 1$, $0 \leq \theta \leq 180^\circ$

8) $\sec^2 \theta - (1 + \sqrt{3}) \tan \theta + \sqrt{3} = 1$, $0 \leq \theta \leq 2\pi$