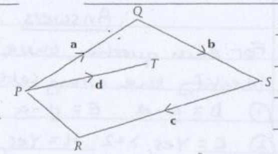
# VECTORS

1. In the diagram, $\overrightarrow{PQ} = \mathbf{a}$, $\overrightarrow{QS} = \mathbf{b}$, $\overrightarrow{SR} = \mathbf{c}$ and $\overrightarrow{PT} = \mathbf{d}$. Find in terms of **a**, **b**, **c** and **d**:




a $\overrightarrow{QT}$
b $\overrightarrow{PR}$
c $\overrightarrow{TS}$
d $\overrightarrow{TR}$

2. In each part, find whether the given vector is parallel to $\mathbf{a} - 3\mathbf{b}$:

a $2\mathbf{a} - 6\mathbf{b}$ b $4\mathbf{a} - 12\mathbf{b}$ c $\mathbf{a} + 3\mathbf{b}$
d $3\mathbf{b} - \mathbf{a}$ e $9\mathbf{b} - 3\mathbf{a}$ f $\frac{1}{2}\mathbf{a} - \frac{2}{3}\mathbf{b}$

3. The non-zero vectors **a** and **b** are not parallel. In each part, find the value of $\lambda$ and the value of $\mu$:

a $\mathbf{a} + 3\mathbf{b} = 2\lambda\mathbf{a} - \mu\mathbf{b}$
b $(\lambda + 2)\mathbf{a} + (\mu - 1)\mathbf{b} = 0$
c $4\lambda\mathbf{a} - 5\mathbf{b} - \mathbf{a} + \mu\mathbf{b} = 0$
d $(1 + \lambda)\mathbf{a} + 2\lambda\mathbf{b} = \mu\mathbf{a} + 4\mu\mathbf{b}$
e $(3\lambda + 5)\mathbf{a} + \mathbf{b} = 2\mu\mathbf{a} + (\lambda - 3)\mathbf{b}$

4. Given that $\mathbf{a} = 9\mathbf{i} + 7\mathbf{j}$, $\mathbf{b} = 11\mathbf{i} - 3\mathbf{j}$ and $\mathbf{c} = -8\mathbf{i} - \mathbf{j}$, find:

a $\mathbf{a} + \mathbf{b} + \mathbf{c}$
b $2\mathbf{a} - \mathbf{b} + \mathbf{c}$
c $2\mathbf{b} + 2\mathbf{c} - 3\mathbf{a}$
(Use column matrix notation in your working.)

5. The points $A$, $B$ and $C$ have coordinates $(3, -1)$, $(4, 5)$ and $(-2, 6)$ respectively, and $O$ is the origin.

Find, in terms of **i** and **j**:

a the position vectors of $A$, $B$ and $C$
b $\overrightarrow{AB}$
c $\overrightarrow{AC}$

Find, in surd form:

d $|\overrightarrow{OC}|$
e $|\overrightarrow{AB}|$
f $|\overrightarrow{AC}|$.

6. Given that $\mathbf{a} = 4\mathbf{i} + 3\mathbf{j}$, $\mathbf{b} = 5\mathbf{i} - 12\mathbf{j}$, $\mathbf{c} = -7\mathbf{i} + 24\mathbf{j}$ and $\mathbf{d} = \mathbf{i} - 3\mathbf{j}$, find a unit vector in the direction of **a**, **b**, **c** and **d**.

7. Find the distance between $A$ and $B$ when they have the following coordinates:

a $A(3, 0, 5)$ and $B(1, -1, 8)$
b $A(8, 11, 8)$ and $B(-3, 1, 6)$
c $A(3, 5, -2)$ and $B(3, 10, 3)$
d $A(-1, -2, 5)$ and $B(4, -1, 3)$

8. Find the modulus of:

a $3\mathbf{i} + 5\mathbf{j} + \mathbf{k}$ b $4\mathbf{i} - 2\mathbf{k}$ c $\mathbf{i} + \mathbf{j} - \mathbf{k}$
d $5\mathbf{i} - 9\mathbf{j} - 8\mathbf{k}$ e $\mathbf{i} + 5\mathbf{j} - 7\mathbf{k}$

9. The points $A$ and $B$ have position vectors $\begin{pmatrix} 2t+1 \\ t+1 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} t+1 \\ 5 \\ 2 \end{pmatrix}$ respectively.

a Find $\overrightarrow{AB}$.
b Find, in terms of $t$, $|\overrightarrow{AB}|$.
c Find the value of $t$ that makes $|\overrightarrow{AB}|$ a minimum.
d Find the minimum value of $|\overrightarrow{AB}|$.

10. The vectors **a** and **b** each have magnitude 3 units, and the angle between **a** and **b** is 60°. Find **a**.**b**.

11. In each part, find the angle between **a** and **b**, giving your answer in degrees to 1 decimal place:

a $\mathbf{a} = 3\mathbf{i} + 7\mathbf{j}$, $\mathbf{b} = 5\mathbf{i} + \mathbf{j}$
b $\mathbf{a} = 2\mathbf{i} - 5\mathbf{j}$, $\mathbf{b} = 6\mathbf{i} + 3\mathbf{j}$
c $\mathbf{a} = \mathbf{i} - 7\mathbf{j} + 8\mathbf{k}$, $\mathbf{b} = 12\mathbf{i} + 2\mathbf{j} + \mathbf{k}$
d $\mathbf{a} = -\mathbf{i} - \mathbf{j} + 5\mathbf{k}$, $\mathbf{b} = 11\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$
e $\mathbf{a} = 6\mathbf{i} - 7\mathbf{j} + 12\mathbf{k}$, $\mathbf{b} = -2\mathbf{i} + \mathbf{j} + \mathbf{k}$
f $\mathbf{a} = 4\mathbf{i} + 5\mathbf{k}$, $\mathbf{b} = 6\mathbf{i} - 2\mathbf{j}$
g $\mathbf{a} = -5\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$, $\mathbf{b} = 2\mathbf{i} - 2\mathbf{j} + 11\mathbf{k}$
h $\mathbf{a} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{b} = \mathbf{i} - \mathbf{j} + \mathbf{k}$

12. Find the value, or values, of $\lambda$ for which the given vectors are perpendicular:

a $3\mathbf{i} + 5\mathbf{j}$ and $\lambda\mathbf{i} + 6\mathbf{j}$
b $2\mathbf{i} + 6\mathbf{j} - \mathbf{k}$ and $\lambda\mathbf{i} - 4\mathbf{j} - 14\mathbf{k}$
c $3\mathbf{i} + \lambda\mathbf{j} - 8\mathbf{k}$ and $7\mathbf{i} - 5\mathbf{j} + \mathbf{k}$
d $9\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$ and $\lambda\mathbf{i} + \lambda\mathbf{j} + 3\mathbf{k}$
e $\lambda\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ and $\lambda\mathbf{i} + \lambda\mathbf{j} + 5\mathbf{k}$

(13) Find a vector equation of the straight line which passes through the point $A$, with position vector $\mathbf{a}$, and is parallel to the vector $\mathbf{b}$:

a) $\mathbf{a} = 6\mathbf{i} + 5\mathbf{j} - \mathbf{k}$, $\mathbf{b} = 2\mathbf{i} - 3\mathbf{j} - \mathbf{k}$

b) $\mathbf{a} = 2\mathbf{i} + 5\mathbf{j}$, $\mathbf{b} = \mathbf{i} + \mathbf{j} + \mathbf{k}$

c) $\mathbf{a} = -7\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}$, $\mathbf{b} = 3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$

d) $\mathbf{a} = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix}$

e) $\mathbf{a} = \begin{pmatrix} 6 \\ -11 \\ 2 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 0 \\ 5 \\ -2 \end{pmatrix}$

(14) Find a vector equation for the line which passes through the points:

a (2, 1, 9) and (4, −1, 8)

b (−3, 5, 0) and (7, 2, 2)

c (1, 11, −4) and (5, 9, 2)

d (−2, −3, −7) and (12, 4, −3)

For questions 15–19, determine whether the lines with the given equations intersect. If they do intersect, find the coordinates of their point of intersection.

(15) $\mathbf{r} = \begin{pmatrix} 2 \\ 4 \\ -7 \end{pmatrix} + t\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 1 \\ 14 \\ 16 \end{pmatrix} + s\begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$

(16) $\mathbf{r} = \begin{pmatrix} 2 \\ 2 \\ -3 \end{pmatrix} + t\begin{pmatrix} 9 \\ -2 \\ -1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} + s\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$

(17) $\mathbf{r} = \begin{pmatrix} 12 \\ 4 \\ -6 \end{pmatrix} + t\begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 8 \\ -2 \\ 6 \end{pmatrix} + s\begin{pmatrix} 2 \\ 1 \\ -5 \end{pmatrix}$

(18) $\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix} + t\begin{pmatrix} 4 \\ 2 \\ 6 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -2 \\ -9 \\ 12 \end{pmatrix} + s\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$

(19) $\mathbf{r} = \begin{pmatrix} 3 \\ -3 \\ 1 \end{pmatrix} + t\begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} + s\begin{pmatrix} 6 \\ -4 \\ 1 \end{pmatrix}$

For questions 20, 21 find, to 1 decimal place, the acute angle between the lines with the given vector equations:

(20) $\mathbf{r} = (2\mathbf{i} + \mathbf{j} + \mathbf{k}) + t(3\mathbf{i} - 5\mathbf{j} - \mathbf{k})$ and $\mathbf{r} = (7\mathbf{i} + 4\mathbf{j} + \mathbf{k}) + s(2\mathbf{i} + \mathbf{j} - 9\mathbf{k})$

(21) $\mathbf{r} = (\mathbf{i} - \mathbf{j} + 7\mathbf{k}) + t(-2\mathbf{i} - \mathbf{j} + 3\mathbf{k})$ and $\mathbf{r} = (8\mathbf{i} + 5\mathbf{j} - \mathbf{k}) + s(-4\mathbf{i} - 2\mathbf{j} + \mathbf{k})$

## Answers

For each question, there is one wrong answer. Identify the wrong letters to form a phrase

(1) $D = \underline{a} - \underline{d}$ $E = \underline{d} - \underline{a}$ $V = \underline{a} + \underline{b} + \underline{c} \div \underline{d}$ $I = \underline{a} + \underline{b} + \underline{c}$ $L = \underline{a} + \underline{b} -$

(2) $C$ = Yes, $\lambda = 2$ $H$ = Yes, $\lambda = 4$ $E$ = Yes, $\lambda = 3$ $A$ = No $P$ = Yes, $\lambda = -$
$L$ = Yes, $\lambda = -3$ $Y$ = No

(3) $H = \lambda = \frac{1}{2}, \mu = -3$ $E = \lambda = -2, \mu = 1$ $A = \lambda = \frac{1}{4}, \mu = 5$
$V = \lambda = 2, \mu = 1$ $E = \lambda = -2, \mu = -1$ $N = \lambda = 4, \mu = 8\frac{1}{2}$

(4) $W = \begin{pmatrix} 12 \\ 3 \end{pmatrix}$ $I = \begin{pmatrix} -1 \\ 16 \end{pmatrix}$ $R = \begin{pmatrix} -21 \\ -29 \end{pmatrix}$ $E = \begin{pmatrix} -5 \\ 13 \end{pmatrix}$

(5) $A = \sqrt{37}$ $L = 3\underline{i} - \underline{j}$ $L = 10\sqrt{2}$ $I = -2\underline{i} + 6\underline{j}$ $G = \sqrt{74}$
$A = 2\sqrt{10}$ $T = \underline{i} + 6\underline{j}$ $O = -5\underline{i} + 7\underline{j}$ $R = 4\underline{i} + 5\underline{j}$

(6) $F = \frac{1}{13}\begin{pmatrix} 5 \\ -12 \end{pmatrix}$ $L = \frac{1}{5}\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ $O = \frac{1}{10}\begin{pmatrix} 1 \\ -3 \end{pmatrix}$ $O = \frac{1}{25}\begin{pmatrix} -7 \\ 24 \end{pmatrix}$ $R = \frac{1}{\sqrt{10}}\begin{pmatrix} 1 \\ -3 \end{pmatrix}$

(7) $D = \sqrt{50}$ $R = \sqrt{30}$ $A = 15$ $P = \sqrt{15}$ $E = \sqrt{14}$

(8) $J = 5\sqrt{3}$ $A = 2\sqrt{5}$ $M = 5\sqrt{2}$ $M = \sqrt{3}$ $E = \sqrt{170}$ $D = \sqrt{35}$

(9) $S = 3$ $A = 2$ $V = \sqrt{2t^2 - 8t + 17}$ $E = \sqrt{2t^2 + 8t - 17}$ $D =$

(10) $O = \frac{9}{2}$ $N = \frac{2}{9}$

(11) $A = 100.9°$ $D = 79.0°$ $V = 70.5°$ $E = 53.7°$ $N = 55.5°$ $T = 11.1°$ $U = 94.8°$ $R = 87.4°$ E

(12) $A = -5$ or $2$ $P = 2\frac{3}{5}$ $A = 3\frac{1}{4}$ $T = -2\frac{1}{2}$ $H = -10$ $Y = 5$

(13) $I = \underline{r} = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix} + t\begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix}$ $N = \underline{r} = \begin{pmatrix} 2 \\ 5 \\ 0 \end{pmatrix} + t\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ $F = \underline{r} = \begin{pmatrix} -7 \\ 6 \\ 2 \end{pmatrix} + t\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$

$L = \underline{r} = \begin{pmatrix} -6 \\ 11 \\ -2 \end{pmatrix} + t\begin{pmatrix} 0 \\ 5 \\ -2 \end{pmatrix}$ $U = \underline{r} = \begin{pmatrix} 6 \\ 5 \\ -1 \end{pmatrix} + t\begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix}$ $X = \underline{r} = \begin{pmatrix} 6 \\ -11 \\ 2 \end{pmatrix} + t\begin{pmatrix} 0 \\ 5 \\ -2 \end{pmatrix}$

(14) $L = \underline{r} = \begin{pmatrix} 2 \\ 1 \\ 9 \end{pmatrix} + t\begin{pmatrix} 10 \\ -3 \\ 2 \end{pmatrix}$ $I = \underline{r} = \begin{pmatrix} 1 \\ 11 \\ -4 \end{pmatrix} + t\begin{pmatrix} 4 \\ -2 \\ 6 \end{pmatrix}$ $M = \underline{r} = \begin{pmatrix} -3 \\ 5 \\ 0 \end{pmatrix} + t\begin{pmatrix} 10 \\ -3 \\ 2 \end{pmatrix}$

$B = \underline{r} = \begin{pmatrix} 2 \\ 1 \\ 9 \end{pmatrix} + t\begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}$ $O = \underline{r} = \begin{pmatrix} -2 \\ -3 \\ -7 \end{pmatrix} + t\begin{pmatrix} 14 \\ 7 \\ 4 \end{pmatrix}$

(15) $Y$ = Yes (8, 7, 1) $B$ = Yes (8, 7, 2)

(16) $A$ = No $M$ = Yes (2, 2, −3)

(17) $A$ = No $T$ = Yes (16, 2, −14)

(18) $O$ = Yes (3, 1, 7) $R$ = Yes (1, 3, 7)

(19) $O$ = No $K$ = Yes (3, 4, 2)

(20) $H = 79.5°$ $E = 79.3°$