1. A curve C has equation

$$y = \sqrt{x-3}\,,\; x > 3.$$

Find an equation of the normal to C at the point where $x = 7$

2. Differentiate each of the following expressions with respect to x, simplifying the final answers as far as possible

a) $y = \left(x^2 - 4\right)^3$

b) $y = x\cos 2x$

c) $y = \dfrac{\sin x}{x}$

3. A curve C has equation

$$y = \sqrt{x^2+1}\,,\; x \in \mathbb{R}.$$

Show that an equation of the normal to C at the point where $x = 1$ is given by

$$y = \sqrt{2}(2-x).$$

4. A curve has equation

$$y = \frac{2x^2+3x}{2x^2-x-6} - \frac{6}{x^2-x-2},\; x \in \mathbb{R},\; 0 < x < 2.$$

a) Show clearly that

$$y = \frac{x+3}{x+1},\; x \in \mathbb{R},\; 0 < x < 2.$$

b) Show further that the equation of the normal to the curve at the point where $x = 1$ passes through the origin.

5. A curve has equation

$$y(y-1) = 5x - 3.$$

Find the gradient at each of the points on the curve where $x = 3$.

6. Differentiate each of the following expressions with respect to x.

a) $y=\left(1-x^2\right)^6$

b) $y=x^3\sin 3x$

c) $y=\dfrac{5x}{x^3+2}$

7. A curve C has equation

$$y=\frac{4x+k}{4x-k},\ x\neq\frac{k}{4}, \text{ where } k \text{ is a constant.}$$

a) Find a simplified expression for $\dfrac{dy}{dx}$, in terms k.

The point P lies on C, where $x=3$.

b) Given that the gradient at P is $-\dfrac{8}{27}$, show that one possible value of k is 48 and find the other.

(1)

$\frac{dy}{dx} = \frac{1}{2}(x-3)^{-\frac{1}{2}}$ OE M1

$\left.\frac{dy}{dx}\right|_{x=7} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ A1 ft

NORMAL GRADIENT $= -4$ A1 ft.

when $x=7$ $y=2$ OR $(7,2)$ B1

$4x + y = 30$ O.E A1

(2)

a) $3(x^2-4)^2 \times 2x$ M1 M1

b) $1 \times \cos 2x + x[-\sin 2x \times 2]$ M1 M1 M1 dep on "correct structure". e.g. $uv' + vu'$

c) $\frac{x\cos x - \sin x}{x^2}$ M1 M1 M1 dep on "correct structure" $\frac{vu' - uv'}{v^2}$

(3)

$\frac{dy}{dx} = \frac{1}{2}(x^2+1)^{-\frac{1}{2}} \times 2x$ M1 M1

$y = 2$ OR $(1, \sqrt{2})$ A1

$\left.\frac{dy}{dx}\right|_{x=1} = \frac{1}{\sqrt{2}}$ OR $\frac{\sqrt{2}}{2}$ A1

NORMAL GRADIENT $-\sqrt{2}$ B1 ft

$y - \sqrt{2} = -\sqrt{2}(x-1)$ M1 ft

CONVINCING SIMPLIFICATION TO $y = \sqrt{2}(2-x)$ A1

4 a) FACTORIZES EITHER DENOMINATOR

$(2x+3)(x-2)$ OR $(x-2)(x+1)$ B1

CANCELS $2x+3$ M1

$$\frac{x(x+1)-6}{(x-2)(x+1)}$$ M1

$$\frac{x^2+x-6}{(x-2)(x+1)}$$ M1

FACTORIZES & CANCELS TO ANSWER

$$\frac{(x+3)\cancel{(x-2)}}{(x+1)\cancel{(x-2)}}$$ A1

b) $\frac{dy}{dx} = \frac{(x+1)\times 1 - (x+3)\times 1}{(x+1)^2} = \frac{-2}{(x+1)^2}$ M3 All marks dependent on quotient rule

"GRAD" $= -\frac{1}{2}$ A1

NORMAL GRAD $= 2$ M1 ft

$y=2$ OR $(1,2)$ B1

$y-2 = $"2"$(x-1)$ M1 ft

$y=2x$ + COMMENT A1

5 $x = \frac{1}{5}y^2 - \frac{1}{5}y + \frac{3}{5}$ M1

$\frac{dx}{dy} = \frac{2}{5}y - \frac{1}{5}$ A1

$\frac{dy}{dx} = \frac{1}{\frac{2}{5}y - \frac{1}{5}}$ OR $\frac{5}{2y-1}$ A1

ALTERNATIVE

$y^2 - y = 5x - 3$

$2y\frac{dy}{dx} - \frac{dy}{dx} = 5$

$(2y-1)\frac{dy}{dx} = 5$

$\frac{dy}{dx} = \frac{5}{2y-1}$

$y^2 - y - 12 = 0$ M1

$(y+3)(y-4)$ M1

$y = 4, -3$ A1

$\frac{5}{7}$, $-\frac{5}{7}$ BOTH A2

(6)

a) $y = (1-x^2)^6$

$$\frac{dy}{dx} = 6(1-x^2)^5 \times (-2x)$$

$$\frac{dy}{dx} = -12x(1-x^2)^5.$$

b) $y = x^3 \sin 3x$

$$\frac{dy}{dx} = 3x^2 \sin 3x + x^3 (3\cos 3x)$$

$$\frac{dy}{dx} = 3x^2(\sin 3x + x\cos 3x)$$

c) $y = \frac{5x}{x^3+2}$

$$\frac{dy}{dx} = \frac{(x^3+2)\times 5 - 5x(3x^2)}{(x^3+2)^2}$$

$$\frac{dy}{dx} = \frac{5x^3+10-15x^3}{(x^3+2)^2} = \frac{10-x^3}{(x^3+2)^2}$$

$$= \frac{10(1-x^3)}{(x^3+2)^2}$$

(7)

a) $y = \frac{4x+k}{4x-k}$

$$\frac{dy}{dx} = \frac{(4x-k)\times 4 - (4x+k)\times 4}{(4x-k)^2} = \frac{\cancel{16x}-4k-\cancel{16x}-4k}{(4x-k)^2}$$

$$\therefore \frac{dy}{dx} = -\frac{8k}{(4x-k)^2}$$

b) $\left.\frac{dy}{dx}\right|_{x=3} = \frac{8}{27}$

$$\Rightarrow \frac{-8k}{(12-k)^2} = \frac{8}{27}$$

$$\Rightarrow -8(12-k)^2 = -216k$$

$$\Rightarrow -(12-k)^2 = -27k$$

$$\Rightarrow [144 - 24k + k^2] = 27k$$

$$\Rightarrow k^2 - 51k + 144 = 0$$

$$\Rightarrow (k-48)(k-3) = 0 \qquad \therefore k = 3, 48$$