Simplify the following algebraic expressions

1) $\frac{2x^2-3x-5}{x^2+x-6} \times \frac{x^2-x-2}{2x^2+x-15}$

   For what values of x is this not valid?

2) $\frac{2x^2-3x-5}{x^2+x-6} \div \frac{x^2-x-2}{2x^2+x-15}$

   For what values of x is this not valid?

3) $\frac{x^2+4x+3}{x^2+5x+6} + \frac{x^2+4x+3}{x^2+3x+2}$

4) $\frac{6x^2+13x-5}{9x^2-6x+1} - \frac{4x^2+20x+25}{6x^2+13x-5}$

5) It is given that $\frac{(27x^3-27x^2+9x-1)(4x^2+4x+1)}{(12x^3+8x^2-x-1)(9x^2-6x+1)} = 1$.

   It is also given that $(3x-1)^3 = 27x^3 - 27x^2 + 9x - 1$
   Use this information to factorise $12x^3 + 8x^2 - x - 1$

Express the following in quotient and remainder form:

6) $\frac{3x^2+2x-1}{x-1}$

7) $\frac{3x^3-4x^2-5x+5}{3x-1}$

8) $\frac{4x^2-8x+1}{2x^2-1}$

9) $\frac{x^4-x^3+2x^2-x-6}{x^2+x-5}$

10) $\frac{x^5-2x^4-x^3+2x^2-x-6}{x^2+x-5}$

Answers

1) $\left(\frac{x+1}{x+3}\right)^2 \quad x \neq 2, \frac{5}{2}$
2) $\left(\frac{2x-5}{x-2}\right)^2 \quad x \neq -3, -1$
3) 2
4) 0
5) $(2x+1)^2(3x-1)$
6) $3x + 5 + \frac{4}{x-1}$
7) $3x - 1 + \frac{3}{3x-1}$
8) $2 + \frac{-8x+3}{2x^2-1}$
9) $x^2 - 2x + 9 + \frac{-20x+39}{x^2+x-5}$
10) $x^3 - 3x^2 + 7x - 20 + \frac{54x-106}{x^2+x-5}$