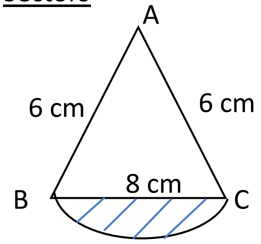
Revision for Tracking Test 2

SECTION A PURE MATHS

1. Sectors



a) Calculate the angle BAC (use the cosine rule).
b) Calculate the arc length BC.
c) Calculate the area of the triangle ABC.
d) Calculate the area of the sector ABC.
e) Find the perimeter of the shaded area to 3 s.f.
f) Find the area of the shaded area to 3 s.f.

2. Inequalities

Solve the following inequalities

a) $2x - 3 \leq 5x + 4$
b) $3 \leq 2x + 1 \leq 5$
c) $x^2 - 5x + 6 \geq 0$ (remember to sketch a graph when solving quadratic inequalities)
d) $2x^2 + 5x - 12 \leq 0$
e) Find the values of k for which $x^2 + kx + k = 0$ has real roots.

3. Coordinate Geometry

A has coordinates (1, -3) and B has co-ordinates (0, 6).
Find

a) The coordinates of the midpoint of AB.
b) The gradient of the line AB.
c) The length of the line segment AB.
d) The equation of the line AB in the form $ax + by + c = 0$
e) The equation of a line which passes through A and is perpendicular to AB in the form $ax + by + c = 0$
f) The equation of the circle which has AB as its diameter.
g) The line AB crosses the x-axis at C and the y-axis at D. Calculate the area of the triangle OCD where O is the point (0,0).

4. Differentiation

A curve has equation $y = \frac{2}{3}x^3 - 3x^2$
Find

a) The gradient of the tangent when $x = -1$
b) The equation of the tangent when $x = 2$
c) Find the x coordinates where the gradient is 8

d) The curve cuts the x axis at C and the y-axis at D. Find the co-ordinates of C and D.

e) Given that the tangents to the curve at P and R are parallel to the line $y - 56x = 6$, find the x coordinate of P and R
f) Find the coordinates of the stationary points of the curve
g) Determine whether the stationery points are a minimum or maximum

5. Trigonometry

Solve the following equations

a)	$2 \sin (x + 10) = 1$	$(0 \leq x \leq 360)$
b)	$\cos^2 x = 0.25$	$(0 \leq x \leq 360)$
c)	$2 \tan (\theta + \frac{\pi}{3}) = 8$	$(0 \leq \theta \leq 2\pi)$
d)	$\cos 2x = -0.9$	$(0 \leq \theta \leq 2\pi)$

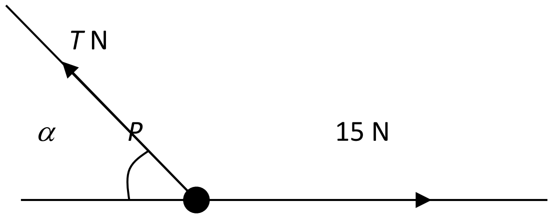
6. Graph Sketching

Sketch showing where each graph cuts the coordinate axes and give the equations of any asymptotes.

a) $y = \frac{1}{x}$
b) $y = \frac{1}{2x}$
c) $y = \frac{1}{x+3}$
d) $y = x^3 + 8$
e) $y = (x + 3)^2 (x - 4)$

SECTION B MECHANICS

1.

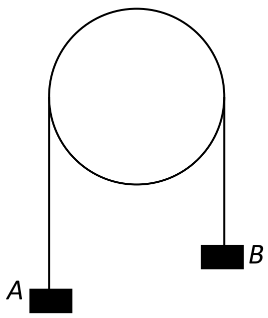


A particle P of mass 2 kg is held in equilibrium under gravity by two light inextensible strings. One string is horizontal and the other is inclined at an angle α to the horizontal. The tension in the horizontal string is 15 N. The tension in the other string is T Newtons.

(*a*) Find the size of the angle α.

(*b*) Find the value of T.

2.



Two particles A and B have masses $3m$ and km respectively, where $k > 3$. They are connected by a light inextensible string which passes over a smooth fixed pulley. The system is released from rest with the string taut and the hanging parts of the string vertical. While the particles are moving freely, A has an acceleration of magnitude $\frac{2}{5}g$.

(*a*) Find, in terms of m and g, the tension in the string.

(*b*) State why B also has an acceleration of magnitude $\frac{2}{5}g$.

(*c*) Find the value of k.

(*d*) State how you have used the fact that the string is light.

3. A parachutist drops from a helicopter H and falls vertically from rest towards the ground. Her parachute opens 2 s after she leaves H and her speed then reduces to 4 m s^{-1}. For the first 2 s her motion is modelled as that of a particle falling freely under gravity. For the next 5 s the model is motion with constant deceleration, so that her speed is 4 m s^{-1} at the end of this period. For the rest of the time before she reaches the ground, the model is motion with constant speed of 4 m s^{-1}.

(*a*) Sketch a speed-time graph to illustrate her motion from H to the ground.

(*b*) Find her speed when the parachute opens.

A safety rule states that the helicopter must be high enough to allow the parachute to open and for the speed of a parachutist to reduce to 4 m s^{-1} before reaching the ground. Using the assumptions made in the above model,

(*c*) find the minimum height of H for which the woman can make a drop without breaking this safety rule.

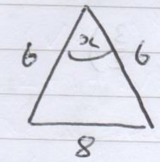
Given that H is 125 m above the ground when the woman starts her drop,

(*d*) find the total time taken for her to reach the ground.

(*e*) State one way in which the model could be refined to make it more realistic.

Please email m.macve@bhasvic.ac.uk if you feel that any of the following answers are incorrect

Revision for TT2

1 a) 

$8^2 = 6^2 + 6^2 - 2 \times 6 \times 6 \cos x$

$\therefore 64 = 72 - 72 \cos x$

$\therefore \cos x = \frac{8}{72}$

$\therefore x = 83.6^\circ$ (3 s.f.)

$= 1.4595 = 1.46$ (3s.f.)

b) Arc $BC = 6 \times 1.46 = 8.7567 = 8.76$ cm (3s f.)

c) Area $= \frac{1}{2} \times 6^2 \times \sin 1.46 = 17.8886 = 17.9$ cm^2 (3 s.f.)

d) Area $= \frac{1}{2} \times 6^2 \times 1.46 = 26.271 = 26.3$ cm^2 (3 s.f.)

e) Perimeter $= 8 + 8.7567 = 16.8$ cm (3 s.f.)

f) Area $= 26.3 - 17.9 = 8.38$ cm^2 (3 s.f.)

2 a) $2x - 3 \leq 5x + 4$

$\therefore -3 \leq 3x + 4$

$\therefore -7 \leq 3x$

$\therefore x \geq -7/3$

b) $3 \leq 2x + 1 \leq 5$

Firstly $3 \leq 2x + 1$

$\therefore 2 \leq 2x$

$\therefore x \geq 1$

Secondly $2x + 1 \leq 5$

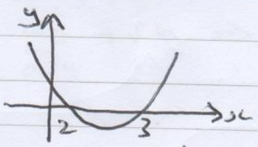
$\therefore 2x \leq 4$

$\therefore x \leq 2$

$\therefore$ overall $1 \leq x \leq 2$

c) $x^2 - 5x + 6 \geq 0$

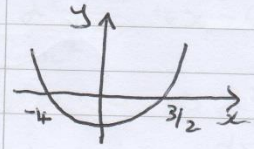
$(x - 3)(x - 2) \geq 0$



$\therefore x \leq 2$ or $x \geq 3$

d) $2x^2 + 5x - 12 \leq 0$

$(2x - 3)(x + 4) \leq 0$



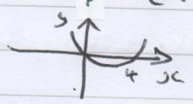
$\therefore -4 \leq x \leq 3/2$

e) For real roots,

discriminant ≥ 0

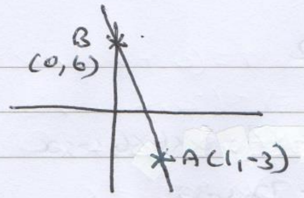
$\therefore k^2 - 4k \geq 0$

$\therefore k(k-4) \geq 0$



$\therefore k \geq 4$ or $k \leq 0$

3.



a) $\left(\frac{1+0}{2}, \frac{-3+6}{2}\right)$

i.e. $\left(\frac{1}{2}, \frac{3}{2}\right)$

b) gradient $= -\frac{9}{1} = -9$

c) length $= \sqrt{1^2+9^2} = \sqrt{82}$

d) $y-6 = -9(x-0)$

i.e. $y-6 = -9x$

i.e. $9x+y-6=0$

e) gradient of perpendicular line is $\frac{1}{9}$

$\therefore$ equation is $y+3 = \frac{1}{9}(x-1)$

i.e. $9y+27 = x-1$

$\therefore x-9y-28=0$

f) centre of circle is $\left(\frac{1}{2}, \frac{3}{2}\right)$

radius of circle is $\frac{1}{2}\sqrt{82}$

$\therefore$ equation of circle is $\left(x-\frac{1}{2}\right)^2+\left(y-\frac{3}{2}\right)^2 = \frac{82}{4}$

i.e. $\left(x-\frac{1}{2}\right)^2+\left(y-\frac{3}{2}\right)^2 = \frac{41}{2}$

g) AB is $9x+y-6=0$

$x=0 \Rightarrow y=6$

$y=0 \Rightarrow x=2/3$

Area $= \frac{1}{2}\times 6\times\frac{2}{3} = 2$

4. $y = \frac{2}{3}x^3 - 3x^2$

$\frac{dy}{dx} = 2x^2 - 6x$

a) $x = -1,\ \frac{dy}{dx} = 2 + 6 = 8$

b) $x = 2,\ \frac{dy}{dx} = 8 - 12 = -4,\quad y = \frac{16}{3} - 12 = \frac{-20}{3}$

$\therefore$ tangent is $y + \frac{20}{3} = -4(x-2)$

i.e. $3y + 20 = -12x + 24$

i.e. $12x + 3y - 4 = 0$

c) $\frac{dy}{dx} = 8 \Rightarrow 2x^2 - 6x = 8$

$\therefore x^2 - 3x = 4$

$\therefore x^2 - 3x - 4 = 0$

$\therefore (x-4)(x+1) = 0$

$\therefore x = 4$ or $x = -1$

d) $x = 0 \Rightarrow y = 0$

$y = 0 \Rightarrow \frac{2}{3}x^3 - 3x^2 = 0$

$\therefore x^2\left(\frac{2}{3}x - 3\right) = 0$

$\therefore x = 0$ or $x = 9/2$

$\therefore C(0,0)\quad D(9/2, 0)$

e) $\frac{dy}{dx} = 56 \Rightarrow 2x^2 - 6x = 56$

$\therefore x^2 - 3x - 28 = 0$

$\therefore (x-7)(x+4) = 0$

$\therefore x = 7$ or $x = -4$

f) $\frac{dy}{dx} = 0 \Rightarrow 2x^2 - 6x = 0$

$\therefore 2x(x-3) = 0$

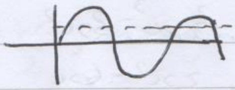
$\therefore x = 0$ or $x = 3$

g) $\frac{d^2y}{dx^2} = 4x - 6$

$x = 0 \Rightarrow \frac{d^2y}{dx^2} = 0 - 6 = -6 < 0 \quad \therefore (0,0)$ is a Maximum

$x = 3 \Rightarrow \frac{d^2y}{dx^2} = 12 - 6 = 6 > 0 \quad \therefore (3,-9)$ is a minimum

5 a) $2\sin(x+10)=1$



$\therefore \sin(x+10°) = \tfrac{1}{2}$

$\therefore x+10° = 30, 150, 390, 410, \ldots$

$\therefore \quad x = 20°, 140°, 380°, 400°, \ldots$

but $0° \le x \le 360°$

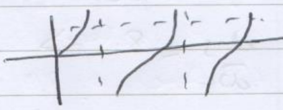
$\therefore x = 20°, 140°$

b) $\cos^2 x = 0.25$

$\therefore \cos x = \frac{1}{2}$ or $\cos x = -\frac{1}{2}$

$\therefore x = 60°, 300°, 120°, 240° \quad (0° \le x \le 360°)$

c) $2\tan\left(\theta+\frac{\pi}{3}\right)=8$

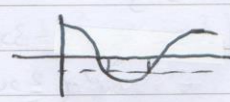


$\therefore \tan(\theta+\pi/3)=4$

$\therefore \quad \theta+\pi/3 = 1.3258, 4.4674, 7.6090 \ldots$

$\therefore \quad \theta = 0.279, 3.42 \quad (0 \le \theta \le 2\pi)$

d) $\cos 2x = -0.9$



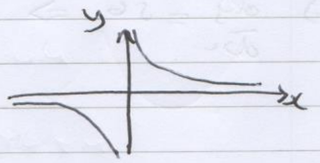
$\therefore \quad 2x = 2.6906, 3.5926,$
$8.9738, 9.8758 \ldots$

$\therefore x = 1.35, 1.80, 4.49, 4.94 \quad (0 \le \theta \le 2\pi)$

6 a)

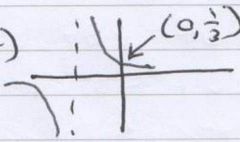
asymptotes $x=0, y=0$

b)



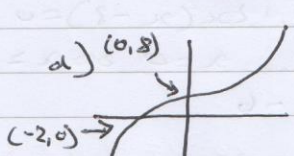
asymptotes $x=0, y=0$

c)

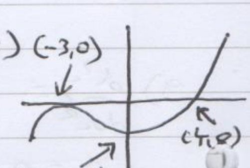


asymptotes $x=3, y=0$

d)

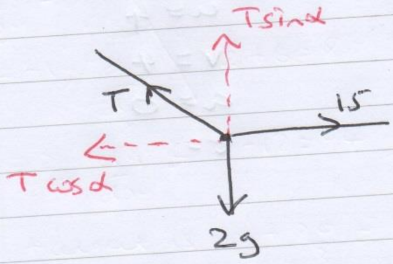


e)



Revision for TT2
Mechanics

(1)



Resolve $\leftrightarrow$ $T\cos\alpha = 15$ — (1)

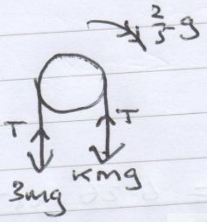
Resolve $\updownarrow$ $T\sin\alpha = 2g$ — (2)

(2) ÷ (1) $\tan\alpha = \frac{2g}{15}$

$\therefore \alpha = 52.573°$

sub in (1) $T = \frac{15}{0.608} = 24.7\text{N}$

(2)



NII on km mass

$kmg - T = km\frac{2}{5}g$ — (1)

NII on 3m mass

$T - 3mg = 3m\frac{2}{5}g$ — (2)

(2) => $T = \frac{6}{5}mg + 3mg = \frac{21}{5}mg$

a)

b) smooth pulley so no friction

c) sub in (1) $kmg - \frac{21}{5}mg = km\frac{2}{5}g$

$\therefore k - \frac{21}{5} = \frac{2}{5}k$

$\therefore \frac{7}{5}k = \frac{21}{5}$

$\therefore k = 3$

d) weight on masses is equal to mass × g

(3) 1st section

$s=$
$u=0$
$v=$
$a=9.8$
$t=2$

2nd section

$s=$
$u=$
$v=4$
$a=$
$t=5$

3rd section

$s=$
$u=4$
$v=4$
$a=0$
$t=$

a) 

b) 1st section, $v = u + at$

$\therefore v = 0 + 19.6$

$\therefore v = 19.6 \text{ ms}^{-1}$

c) 2nd section $u = 19.6$

$s = \frac{u+v}{2} t$

$\therefore s = \frac{19.6 + 4}{2} \times 5 = 59\text{m}$

d) 1st section $s = \frac{u+v}{2} t$

$\therefore s = \frac{0+19.6}{2} \times 2 = 19.6\text{m}$

3rd section, $s = 125 - 59 - 19.6 = 46.4\text{m}$

$s = \frac{u+v}{2} t$

$\therefore 46.4 = 4t$

$\therefore t = 11.6\text{s}$

$\therefore$ total time $= 2 + 5 + 11.6 = 18.6\text{s}$

e) The parachute doesn't open instantaneously so there is a period of time when she is decelerating more slowly