1	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	States that: $A(x-4)(3x+1)+B(3x+1)+C(x-4)(x-4)\equiv 18x^2-98x+78$	**M1**	2.2a	7th Decompose algebraic fractions into partial fractions – repeated factors.
	Further states that: $A(3x^2-11x-4)+B(3x+1)+C(x^2-8x+16)\equiv 18x^2-98x+78$	**M1**	1.1b	
	Equates the various terms. Equating the coefficients of x^2: $3A+C=18$ Equating the coefficients of x: $-11A+3B-8C=-98$ Equating constant terms: $-4A+B+16C=78$	**M1**	2.2a	
	Makes an attempt to manipulate the expressions in order to find A, B and C. Obtaining two different equations in the same two variables would constitute an attempt.	**M1**	1.1b	
	Finds the correct value of any one variable: either $A = 4$, $B = -2$ or $C = 6$	**A1**	1.1b	
	Finds the correct value of all three variables: $A = 4$, $B = -2$, $C = 6$	**A1**	1.1b	
				(6 marks)

Notes

Alternative method

Uses the substitution method, having first obtained this equation:

$A(x-4)(3x+1)+B(3x+1)+C(x-4)(x-4)\equiv 18x^2-98x+78$

Substitutes $x = 4$ to obtain $13B = -26$

Substitutes $x=-\frac{1}{3}$ to obtain $\frac{169}{9}C=\frac{338}{3}\Rightarrow C=\frac{1014}{169}=6$

Equates the coefficients of x^2: $3A+C=18$

Substitutes the found value of C to obtain $3A = 12$

2	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Differentiates 4^x to obtain $4^x \ln 4$	**M1**	1.1b	7th Differentiate simple functions defined implicitly.
	Differentiates $2xy$ to obtain $2x\dfrac{dy}{dx}+2y$	**M1**	2.2a	
	Rearranges $4^x \ln 4 = 2x\dfrac{dy}{dx}+2y$ to obtain $\dfrac{dy}{dx}=\dfrac{4^x \ln 4 - 2y}{2x}$	**A1**	1.1b	
	Makes an attempt to substitute (2, 4)	**M1**	1.1b	
	States fully correct final answer: $4\ln 4 - 2$ Accept $\ln 256 - 2$	**A1**	1.1b	
				(5 marks)

Notes

3	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Correctly states $\cos(5x+2x) \equiv \cos 5x \cos 2x - \sin 5x \sin 2x$	M1	1.1b	6th Integrate using trigonometric identities.
	Correctly states $\cos(5x-2x) \equiv \cos 5x \cos(-2x) - \sin 5x \sin(-2x)$ or states $\cos(5x-2x) \equiv \cos 5x \cos(2x) + \sin 5x \sin(2x)$	M1	1.1b	
	Adds the two above expressions and states $\cos 7x + \cos 3x \equiv 2\cos 5x \cos 2x$	A1	1.1b	
		(3)		
(b)	States that $\int(\cos 5x \cos 2x)\,\mathrm{d}x = \frac{1}{2}\int(\cos 7x + \cos 3x)\,\mathrm{d}x$	M1	2.2a	6th Integrate functions of the form f($ax+b$).
	Makes an attempt to integrate. Changing cos to sin constitutes an attempt.	M1	1.1b	
	Correctly states the final answer $\frac{1}{14}\sin 7x + \frac{1}{6}\sin 3x + C$ o.e.	A1	1.1b	
		(3)		
				(6 marks)

Notes

(b) Student does not need to state '+C' to be awarded the first method mark. Must be stated in the final answer.

4	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Makes an attempt to substitute $t = 0$ into $\mathrm{T}(t) = T_R + (90 - T_R)\mathrm{e}^{-\frac{1}{20}t}$. For example, $\mathrm{T}(t) = T_R + (90 - T_R)\mathrm{e}^{0}$ or $\mathrm{T}(t) = T_R + (90 - T_R)$ is seen.	**M1**	3.1a	6th Set up and use exponential models of growth and decay.
	Concludes that the T_R terms will always cancel at $t = 0$, therefore the room temperature does not influence the initial coffee temperature.	**B1**	3.5a	
		(2)		
(b)	Makes an attempt to substitute $T_R = 20$ and $t = 10$ into $\mathrm{T}(t) = T_R + (90 - T_R)\mathrm{e}^{-\frac{1}{20}t}$. For example, $\mathrm{T}(10) = 20 + (90 - 20)\mathrm{e}^{-\frac{1}{20}(10)}$ is seen.	**M1**	1.1b	6th Set up and use exponential models of growth and decay.
	Finds $\mathrm{T}(10) = 62.457...°\mathrm{C}$. Accept awrt 62.5°.	**A1**	1.1b	
		(2)		
				(4 marks)

Notes

5	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Begins the proof by assuming the opposite is true. ‘Assumption: there exists a number n such that n is odd and $n^3 + 1$ is also odd.’	**B1**	3.1	7th Complete proofs using proof by contradiction.
	Defines an odd number. ‘Let $2k + 1$ be an odd number.’	**B1**	2.2a	
	Successfully calculates $(2k+1)^3+1$ $(2k+1)^3+1 \equiv \left(8k^3+12k^2+6k+1\right)+1 \equiv 8k^3+12k^2+6k+2$	**M1**	1.1b	
	Factors the expression and concludes that this number must be even. $8k^3+12k^2+6k+2 \equiv 2\left(4k^3+6k^2+3k+1\right)$ $2\left(4k^3+6k^2+3k+1\right)$ is even.	**M1**	1.1b	
	Makes a valid conclusion. This contradicts the assumption that there exists a number n such that n is odd and $n^3 + 1$ is also odd, so if n is odd, then $n^3 + 1$ is even.	**B1**	2.4	
				(5 marks)

Notes

Alternative method

Assume the opposite is true: there exists a number n such that n is odd and $n^3 + 1$ is also odd. (**B1**)

If $n^3 + 1$ is odd, then n^3 is even. (**B1**)

So 2 is a factor of n^3. (**M1**)

This implies 2 is a factor of n. (**M1**)

This contradicts the statement n is odd. (**B1**)

6	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Recognises that the identity $\sin^2 t + \cos^2 t \equiv 1$ can be used to find the cartesian equation.	**M1**	2.2a	6th Convert between parametric equations and cartesian forms using trigonometry.
	States $\sin t = \frac{y}{2}$ or $\sin^2 t = \frac{y^2}{4}$ Also states $\cos^2 t = \frac{1}{x-1}$	**M1**	1.1b	
	Substitutes $\sin^2 t = \frac{y^2}{4}$ and $\cos^2 t = \frac{1}{x-1}$ into $\sin^2 t + \cos^2 t \equiv 1$ $\frac{y^2}{4} + \frac{1}{x-1} = 1 \Rightarrow \frac{y^2}{4} = \frac{x-2}{x-1}$	**M1**	1.1b	
	Solves to find $y = \sqrt{\frac{4x-8}{x-1}}$, accept $y = \sqrt{\frac{8-4x}{1-x}}$, $x < 1$ or $x \geqslant 2$	**A1**	1.1b	
				(4 marks)

Notes

7	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Understands that for the series to be convergent $\|r\|<1$ or states $\|-4x\|<1$	M1	2.2a	6th Understand convergent geometric series and the sum to infinity.
	Correctly concludes that $\|x\|<\frac{1}{4}$. Accept $-\frac{1}{4}<x<\frac{1}{4}$	A1	1.1b	
		(2)		
(b)	Understands to use the sum to infinity formula. For example, states $\frac{1}{1+4x}=4$	M1	2.2a	5th Understand sigma notation.
	Makes an attempt to solve for x. For example, $4x=-\frac{3}{4}$ is seen.	M1	1.1b	
	States $x=-\frac{3}{16}$	A1	1.1b	
		(3)		
				(5 marks)

Notes

8	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Finds $f(1.9) = -0.2188...$ and $f(2.0) = (+)0.1606...$	M1	1.1b	5th Use a change of sign to locate roots.
	Change of sign and continuous function in the interval $[1.9, 2.0] \Rightarrow$ root	A1	2.4	
		(2)		
(b)	Makes an attempt to differentiate $f(x)$	M1	2.2a	6th Solve equations approximately using the Newton-Raphson method.
	Correctly finds $f'(x) = -9\sin^2 x\cos x + \sin x$	A1	1.1b	
	Finds $f(1.95) = -0.0348...$ and $f'(1.95) = 3.8040...$	M1	1.1b	
	Attempts to find x_1 $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \Rightarrow x_1 = 1.95 - \frac{-0.0348...}{3.8040...}$	M1	1.1b	
	Finds $x_1 = 1.959$	A1	1.1b	
		(5)		
				(7 marks)

Notes

(a) Minimum required is that answer states there is a sign change in the interval and that this implies a root in the given interval.

9	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	States $-a+b=10$ and $7a+5b=2$	**M1**	2.2a	6th Solve geometric problems using vectors in 3 dimensions
	Makes an attempt to solve the pair of simultaneous equations. Attempt could include making a substitution or multiplying the first equation by 5 or by 7.	**M1**	1.1b	
	Finds $a = -4$	**A1**	1.1b	
	Find $b = 6$	**A1**	1.1b	
	States $-2abc = -96$	**M1**	2.2a	
	Finds $c = -2$	**A1**	1.1b	
				(6 marks)

Notes

10	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Begins the proof by assuming the opposite is true. 'Assumption: there exist positive integer solutions to the statement $x^2 - y^2 = 1$'	**B1**	3.1	7th Complete proofs using proof by contradiction.
	Sets up the proof by factorising $x^2 - y^2$ and stating $(x - y)(x + y) = 1$	**M1**	2.2a	
	States that there is only one way to multiply to make 1: $1 \times 1 = 1$ and concludes this means that: $x - y = 1$ $x + y = 1$	**M1**	1.1b	
	Solves this pair of simultaneous equations to find the values of x and y: $x = 1$ and $y = 0$	**M1**	1.1b	
	Makes a valid conclusion. $x = 1$, $y = 0$ are not both positive integers, which is a contradiction to the opening statement. Therefore there do not exist positive integers x and y such that $x^2 - y^2 = 1$	**B1**	2.4	
				(5 marks)

Notes

11	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Understands the need to complete the square, and makes an attempt to do this. For example, $(x-4)^2$ is seen.	**M1**	2.2a	6th Find the domain and range of inverse functions.
	Correctly writes $g(x)=(x-4)^2-9$	**A1**	1.1b	
	Demonstrates an understanding of the method for finding the inverse is to switch the *x* and *y*. For example, $x=(y-4)^2-9$ is seen.	**B1**	2.2a	
	Makes an attempt to rearrange to make *y* the subject. Attempt must include taking the square root.	**M1**	1.1b	
	Correctly states $g^{-1}(x)=\sqrt{x+9}+4$	**A1**	1.1b	
	Correctly states domain is $x > -9$ and range is $y > 4$	**B1**	3.2b	
				(6 marks)

Notes

12	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	States that: $A(4-x)(x+5)+B(x-3)(x+5)+C(x-3)(4-x)\equiv 4x^2+x-23$	**M1**	2.2a	6th Decompose algebraic fractions into partial fractions − three linear factors.
	Further states that: $A(-x^2-x+20)+B(x^2+2x-15)+C(-x^2+7x-12)\equiv 4x^2+x-23$	**M1**	1.1b	
	Equates the various terms. Equating the coefficients of x^2: $-A+B-C=4$ Equating the coefficients of x: $-A+2B+7C=1$ Equating constant terms: $20A-15B-12C=-23$	**M1***	2.2a	
	Makes an attempt to manipulate the expressions in order to find *A*, *B* and *C*. Obtaining two different equations in the same two variables would constitute an attempt.	**M1***	1.1b	
	Finds the correct value of any one variable: either $A = 2$, $B = 5$ or $C = -1$	**A1***	1.1b	
	Finds the correct value of all three variables: $A = 2$, $B = 5$, $C = -1$	**A1**	1.1b	
				(6 marks)

Notes

Alternative method

Uses the substitution method, having first obtained this equation:

$A(4-x)(x+5)+B(x-3)(x+5)+C(x-3)(4-x)\equiv 4x^2+x-23$

Substitutes $x = 4$ to obtain $9B = 45$ (**M1**)

Substitutes $x = 3$ to obtain $8A = 16$ (**M1**)

Substitutes $x = -5$ to obtain $-72C = 72$ (**A1**)

13	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Finds $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + 12x - 12$	**M1**	1.1b	7th Use second derivatives to solve problems of concavity, convexity and points of inflection.
	Finds $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 6x + 12$	**M1**	1.1b	
	States that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 6x + 12 \leq 0$ for all $-5 \leqslant x \leqslant -3$ and concludes this implies C is concave over the given interval.	**B1**	3.2a	
		(3)		
(b)	States or implies that a point of inflection occurs when $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 0$	**M1**	3.1a	7th Use second derivatives to solve problems of concavity, convexity and points of inflection.
	Finds $x = -2$	**A1**	1.1b	
	Substitutes $x = -2$ into $y = x^3 + 6x^2 - 12x + 6$, obtaining $y = 46$	**A1**	1.1b	
		(3)		
				(6 marks)

Notes

14	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Makes an attempt to find $\int \sin 4x(1-\cos 4x)^3 \, dx$. Raising the power by 1 would constitute an attempt.	**M1**	2.2a	6th Integrate using the reverse chain rule.
	States a fully correct answer $\int \sin 4x(1-\cos 4x)^3 \, dx = \frac{1}{16}(1-\cos 4x)^4 + C$	**M1**	2.2a	
	Makes an attempt to substitute the limits $\frac{1}{16}\left[(1-0)^4 - \left(1-\frac{1}{2}\right)^4\right]$	**M1 ft**	1.1b	
	Correctly states answer is $\frac{15}{256}$	**A1 ft**	1.1b	
				(4 marks)

Notes

Student does not need to state '+C' to be awarded the second method mark.

Award ft marks for a correct answer using an incorrect initial answer.

15	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Makes an attempt to set up a long division. For example, $2x^2 - x - 1 \overline{)4x^2 - 4x - 9}$ is seen.	M1	2.2a	7th Expand rational functions using partial fraction decomposition.
	Long division completed so that a 2 is seen in the quotient and a remainder of $-2x - 7$ is also seen. $\begin{array}{r} 2 \\ 2x^2 - x - 1 \overline{)4x^2 - 4x - 9} \\ \underline{4x^2 - 2x - 2} \\ -2x - 7 \end{array}$	M1	1.1b	
	States $B(x-1) + C(2x+1) \equiv -2x - 7$	M1	2.2a	
	Either equates variables or makes a substitution in an effort to find B or C.	M1	2.2a	
	Finds $B = 4$	A1	1.1b	
	Finds $C = -3$	A1	1.1b	
		(6)		
(b)	Correctly writes $4(2x+1)^{-1}$ or $4(1+2x)^{-1}$ as $4\left(1 + (-1)(2x) + \dfrac{(-1)(-2)(2)^2 x^2}{2} + \ldots\right)$	M1 ft	2.2a	6th Understand the binomial theorem for rational n.
	Simplifies to obtain $4 - 8x + 16x^2 + \ldots$	A1 ft	1.1b	
	Correctly writes $\dfrac{-3}{x-1}$ as $\dfrac{3}{1-x}$	M1 ft	2.2a	
	Correctly writes $3(1-x)^{-1}$ as $3\left(1 + (-1)(-x) + \dfrac{(-1)(-2)(-1)^2(-x)^2}{2} + \ldots\right)$	M1 ft	2.2a	
	Simplifies to obtain $3 + 3x + 3x^2 + \ldots$	A1 ft	1.1b	
	States the correct final answer: $9 - 5x + 19x^2$	A1 ft	1.1b	
		(6)		

(c)	The expansion is only valid for $\lvert x \rvert < \frac{1}{2}$	**B1**	3.2b	6th Understand the conditions for validity of the binomial theorem for rational n.
		(1)		
(13 marks)				

Notes

(a) Alternative method.

Writes the RHS as a single fraction.

Obtains $4x^2 - 4x - 9 = A(2x+1)(x-1) + B(x-1) + C(2x+1)$

Substitutes $x = 1$ to obtain $C = -3$

Substitutes $x = -\frac{1}{2}$ to obtain $B = 4$

Compares coefficients of x^2 to obtain $A = 2$

(b) Award all 6 marks for a correct answer using their incorrect values of *A*, *B* and/or *C* from part **a**.

16	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	States $\dfrac{dh}{dt}=\dfrac{dV}{dt}\times\dfrac{dh}{dV}$	**M1**	3.1b	8th Solve differential equations in a range of contexts.
	Deduces that $V=\pi r^2 h=1600\pi h$	**M1**	3.1b	
	Finds $\dfrac{dV}{dh}=1600\pi$ and/or $\dfrac{dh}{dV}=\dfrac{1}{1600\pi}$	**M1**	1.1b	
	States $\dfrac{dV}{dt}=4000\pi-50\pi h$	**M1**	3.1b	
	Makes an attempt to find $\dfrac{dh}{dt}=\left(4000\pi-50\pi h\right)\times\dfrac{1}{1600\pi}$	**M1**	1.1b	
	Shows a clear logical progression to state $160\dfrac{dh}{dt}=400-5h$	**A1**	1.1b	
		(6)		
(b)	Separates the variables $\displaystyle\int\left(\frac{1}{400-5h}\right)dh=\int\frac{1}{160}\,dt$	**M1**	2.2a	8th Solve differential equations in a range of contexts.
	Finds $-\dfrac{1}{5}\ln\left(400-5h\right)=\dfrac{t}{160}+C$	**A1**	1.1b	
	Uses the fact that t = 0 when h = 50 m to find C $C=-\dfrac{1}{5}\ln\left(150\right)$	**M1**	1.1b	
	Substitutes h = 60 into the equation $-\dfrac{1}{5}\ln\left(400-300\right)=\dfrac{t}{160}-\dfrac{1}{5}\ln\left(150\right)$	**M1**	3.1b	
	Uses law of logarithms to write $\dfrac{1}{5}\ln\left(150\right)-\dfrac{1}{5}\ln\left(100\right)=\dfrac{t}{160}$ $\Rightarrow\dfrac{1}{5}\ln\left(\dfrac{150}{100}\right)=\dfrac{t}{160}$	**M1**	2.2a	
	States correct final answer $t=32\ln\left(\dfrac{3}{2}\right)$ minutes.	**A1**	1.1b	
		(6)		
				(12 marks)

Notes